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Chaotic Electron Motion Caused by Sidebands in Free Electron Lasers

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The electron dynamics in a Free Electron Laser (FEL) is studied in the case when the radiation field contains many modes. This situation arises when unstable modes (sidebands) are excited during operation. It is observed that when the strength of these sidebands exceeds certain levels, the electron motion becomes chaotic. This may lead to extensive particle detrapping and loss of amplification for the FEL signal. The threshold for the onset of stochastic electron motion is computed. The evolution of the trapped electron distribution exhibits a diffusive behavior. The rate of particle detrapping is parameterized by the diffusion coefficient D in action space. The e-folding length for the number of trapped electrons is parametrized by J_s^2/D where J_s is the action at the separatrix. It is found that the diffusion rates are connected to the type of the sideband spectrum. The diffusion coefficient is always proportional to the ratio of the sideband power in all

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19. ABSTRACTS (Continued)

frequencies to the power of the carrier signal. The coefficient of the proportionality however scales differently on the FEL parameters for each of the three spectral categories: a narrow, a broad discrete and a broad continuous spectrum. The diffusion coefficient is computed analytically for the last two cases and is in good agreement with numerical results. The narrow spectrum yields the highest and the broad continuous the lowest diffusion rates under constant sideband power. It is also found that, in all cases, the diffusion length, measured in wiggler periods, is independent of the electron energy γ.

CONTENTS

CHAOTIC ELECTRON MOTION CAUSED BY SIDEBANDS IN FREE ELECTRON LASERS

I. INTRODUCTION

Multifrequency effects in Free Electron Lasers (FELs) become increasingly important as progress is made towards high power operation. Growth of parasitic frequencies (sidebands[1-5]) has been predicted theoretically and has been observed in experiments[6,7] as well as in simulations[8-11] with either constant or tapered wigglers[11]. The efficiency for the carrier signal is reduced and the optical quality is degraded as power is channeled into frequencies apart from the intended operation frequency. Another potential hazard that has attracted little attention so far is the onset of chaotic electron motion caused by the presence of even a single frequency sideband. This may lead to extensive particle detrapping and premature loss of the amplification for **all** the radiation modes independent of frequency.

Two of the main issues concerning FEL operation are: (a) whether unstable parasitic frequencies exist that can grow to significant amplitude and (b) what is the effect of potentially unstable modes on the trapped electron trajectories. Considerable attention has been devoted to the linear stability issue. The gain for small sideband signal has been computed analytically[1-5] invoking either ensemble averaging over single particle trajectories or solutions of the perturbed kinetic equation for the distribution function. Initial results, obtained for particles localized near the bottom of the ponderomotive well, and, in particular, more recent results including all trapped and untrapped particles[2] with arbitrary distributions, have demonstrated that every nontrivial distribution $df_0/dJ \neq 0$ is unstable to sideband growth.

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Given that sidebands cannot be eliminated, the growth of the unstable modes to a finite amplitude may have serious effects on the unperturbed trajectories. It has been known that stochastic behavior[12] is an intrinsic property of perturbed Hamiltonian systems[13,14]. Accordingly, the electron motion in a FEL will become chaotic when the sideband amplitude exceeds a certain threshold. This, in turn, will result in significant electron detrapping. Since it is the deceleration of the trapped electron bucket that provides the energy for the radiation in case of tapered wigglers, detrapping will cause loss of amplification for the FEL signal.

In the present work we investigate the nonlinear effects caused by sidebands. The threshold for stochasticity, above which unbound chaotic motion occurs, is determined. Once the stochastic transition takes place, the action J, a constant of motion in the unperturbed system, changes in a random manner. The ensemble average $\langle \Delta X^2 \rangle$ of any physical quantity X is described by a diffusion equation. Diffusion of the action invariant provides a measure of the leakage rate across the separatrix. If D is the effective diffusion coefficient in action space then the diffusion length $L_d = J_s^2/D$, where J_s is the action at the separatrix, signifies the length over which approximately half of the deeply trapped particles get detrapped. We show that a single frequency sideband at a modest fraction of the carrier amplitude suffices to spread irregular motion over a significant fraction of the trapped particle domain. However, given that the interaction time of an electron in a FEL is short, we are mainly concerned on **how fast** this diffusion occurs. The diffusion rate increases and the diffusion length L_d decreases with increasing sideband amplitude(s). Thus, a critical sideband level a_c can be defined above which the diffusion length L_d

becomes shorter than the wiggler length L_w. Obviously the power level for the sidebands in a FEL cannot exceed a_c, otherwise, extensive diffusion and premature detrapping will occur. On the other hand, enough electrons can remain trapped during the interaction period even though their motion has turned stochastic, because we find that usually the critical amplitude a_c is much larger than the threshold for stochasticity α_s.

A clear-cut relation between the diffusion rate under constant total sideband power and the type of the excited sideband spectrum is discovered. More specifically we observe three regimes in the simulation parameters defining the sideband spectrum, corresponding to a narrow, a wide discrete and a wide continuous spectrum. The transition from one spectral type to another is accompanied by an abrupt change in the diffusion rates. In all cases we find the diffusion coefficient proportional to the ratio of the total power in the sidebands to the FEL carrier power. The coefficients of this proportionality depend on the spectral type. A general conclusion is that the diffusion rate under constant sideband power ratio decreases with increasing number of spectral components. The diffusion rate for a single sideband frequency exceeds that of a broad continuous spectrum by orders of magnitude while a broad discrete spectrum causes intermediate diffusion rates.

For practical purposes we measure the diffusion length in terms of the number of wiggler periods, $N_d = L_d/\lambda_w$, while $\langle \Delta J^2 \rangle$ is normalized to the action J_s at the (unperturbed) separatrix. We compute the normalized diffusion coefficient D analytically for the cases of broad discrete and continuous spectra. In the latter case the quasilinear diffusion coefficient in action space $D_q(J)$ is obtained in closed form. This expression for $D_q(J)$ is quite general, valid for any choice of

unperturbed Hamiltonian $H_0(J)$. The analysis also shows that the normalized diffusion coefficient does not depend on the beam energy γ_r. The numerical results agree well with the theory.

We evaluate the loss of trapped particles for typical short wavelength FEL parameters. We find that a single frequency sideband with a sideband to carrier power ratio of $\lesssim 1$ can cause half of the particles to detrap over 100 wiggler periods; we have observed total loss of trapped particles for power ratios of ≈ 1. In cases of wide but discrete sideband spectrum the diffusion length becomes comparable to the wiggler length only at large power ratios ($\gtrsim 1$). The case of a wide continuous spectrum seems to cause insignificant electron detrapping for the same parameters as above; the typical diffusion length is of the order of 1000 wiggler periods for sideband to carrier power ratios of 1.

In our investigation we have assumed all electromagnetic fields as given. The changes in the particle trajectories are decoupled from the evolution of the fields. At the expense of self-consistency we are able to analyze the situation theoretically and determine the scaling of the diffusion rates on the various FEL parameters. Deterioration in the extraction efficiency has been observed in self-consistent numerical simulations of high power FEL oscillators[10] with high level sideband excitation. The gain per pass in a tapered wiggler is progressively limited as the sideband power goes up and the rate of electron detrapping is accelerated. In an untapered wiggler, on the other hand, particle detrapping is not so important for the main signal efficiency. The total extraction efficiency may actually increase with the sidebands since there are more modes to channel the electron beam energy into.

The remainder of this paper is organized as follows. In Sec. II we construct our analytic model for the study of the stochastic diffusion and discuss the various approximations. To elucidate the analysis we start with a single sideband mode and give a sketchy description of how this can lead to electron detrapping. In Sec. III we examine the structure of the phase space for a monochromatic sideband in detail, using canonical formalism. The threshold for the stochastic transition and the extent of the chaotic regime in phase space are obtained in Sec. IV. In Sec. V the diffusion rate caused by a single sideband mode is examined in connection with the various FEL parameters. In Sec. VI the study is extended to broad (multifrequency) sideband spectra. A distinction is drawn between continuous and discrete spectra. Subsection VI.a covers the case of a broad discrete spectrum and the related diffusion coefficient. Subsection VI.b deals with a broad continuous spectrum and the corresponding quasilinear diffusion coefficient. In Sec. VII the theoretical models are compared with numerical results. The differences in the induced diffusion rates among the three different types of spectra are emphasized. The reduction in the extraction efficiency in a tapered wiggler FEL is computed as a function of the diffusion coefficient. Results and conclusions are summarized in Sec. VIII.

II. GENERAL CONSIDERATIONS

We consider relativistic electrons streaming along the z-direction through the static magnetic wiggler and the radiation fields of the carrier and the sideband. We take all fields to be circularly polarized and of constant amplitude. To simplify the analysis and make the underlying ideas clearer we start out with monochromatic waves for the carrier and the sideband. The total vector potential is then,

$$A\ (z,t) = \tag{1}$$
$$\frac{1}{2}\left[(e_x - ie_y)A_w e^{i\phi_w} - (e_x + ie_y)A_r e^{i(k_r z - w_r t)} - (e_x + ie_y)A_s e^{i(k_s z - w_s t)}\right] + cc\ ,$$

where the subscripts w, r, and s stand for wiggler, carrier and sideband respectively. We assume that all waves propagate with the speed of light c, ignoring the small correction of order ω_p^2/ω_r^2 from the dielectric contribution of the beam. Electrostatic contributions to the fields are neglected for operation in the Compton regime. The phase of the wiggler is given by $\phi_w(z) = \int^z k_w(z')dz'$, where the wave number $k_w(z)$ may change slowly in z on a scale length much longer than the wiggler wavelength $\lambda_w = 2\pi/k_w$. The main signal wave number k_r is doubly Doppler upshifted from the wiggler wavenumber k_w,

$$k_r = 2\gamma_z^2 k_w\ , \tag{2}$$

with the upshifting factor $\gamma_z = (1 - \beta_r^2)^{-1/2}$ and $\beta_r = \omega_r/c(k_r + k_w)$.

We have ignored variations in the x- and y-directions. Increased number of dimensions is known to facilitate the transition to chaotic

motion. Therefore, the threshold for stochasticity for variations in the z-direction only will be useful in providing a neccessary condition to avoid fast large-scale diffusion. Slow diffusion due to higher dimensionality will in fact persist for the real system below this threshold. As far as particle detrapping is concerned, three dimensional effects are comparatively insignificant, provided that the dependence on x and y is adiabatic. This requires that the frequency of the betatron oscillation, caused by the transverse field gradients[15], be small compared to the electron synchrotron frequency in the ponderomotive bucket.

We have also assumed that the radiation amplitudes remain constant. In case of fast growth rate of the carrier amplitude the particle trajectories are not analytically tractable, even in the absence of sidebands. It is generally expected that the fraction of trapped particles decreases with decreasing carrier amplitude. Therefore the spreading of the radiation beam due to diffraction[16,17] can also cause detrapping by reducing the carrier amplitude a_r. This detrapping mechanism is independent of the diffusive detrapping caused by sidebands that is examined here.

Normalizing the time t to ω_r^{-1}, the length z to k_r^{-1}, the mass to m_e and the vector potentials according to $a_i = |e|A_i/m_e c^2$ the dimensionless Hamiltonian describing the electron motion in the fields of Eq. (1) is,

$$H = \left[\mu^2 + p_z^2 - 2\Lambda\, a_w a_r \cos(\phi_w + k_r z - \omega_r t) - 2\Lambda\, a_w a_s \cos(\phi_w + k_s z - \omega_s t)\right]^{1/2} \tag{3}$$

$$\mu^2 = 1 + M(a_w^2 + a_r^2 + a_s^2)\ ,$$

with M = 1 and $\Lambda = 1$. Eq. (3) also describes the fast-time averaged Hamiltonian for a linearly polarized wiggler by setting M = 1/2, $\Lambda = [J_o(\zeta) - J_1(\zeta)]/2$ and $\zeta = a_w^2/(4 + 2a_w^2)$.

The terms proportional to $a_w a_r$ and $a_w a_s$ are the ponderomotive potentials due to the combined action of the wiggler with the main signal and the sideband respectively. The resonant velocities for each ponderomotive potential are given by $\beta_i = k_i/(k_i + k_w)$ corresponding to resonant energies,

$$\gamma_i = \left(\frac{\mu^2}{1 - \beta_i^2}\right)^{1/2}, \qquad i = r, s. \tag{4}$$

In the vicinity of γ_i the motion of the electrons is determined by the corresponding resonant term inside (3). We may drop the nonresonant term for small radiation amplitudes and linearize (3) for small excursions $\delta\gamma$ around γ_i. From the resulting pendulum equation we find that trapped electrons will undergo oscillations of frequency ω_b around γ_i, forming islands of width $\delta\gamma_i$ in phase space, where ω_b and $\delta\gamma_i$ are given by,

$$\omega_b = \frac{1}{\gamma_i^2}\left(a_w a_i \mu^2\right)^{1/2}, \qquad \delta\gamma_i = \gamma_i\left(\frac{a_w a_r}{\mu^2}\right)^{1/2}. \tag{5}$$

We call these islands, due to the direct wave-particle resonances, primary islands.

Roughly speaking, irregular motion breaks out as a result of nearby island overlapping[14]. The amplitudes a_r, a_s must increase to the point where,

$$\delta\gamma_r + \delta\gamma_s \geq |\gamma_r - \gamma_s|, \tag{6}$$

for an overlapping between the two primary islands to take place. The difference in resonant energies $\Delta\gamma = \gamma_r - \gamma_s$ is given by $\Delta\gamma \simeq \gamma_z\gamma_r^2\ \Delta\beta$ where $\Delta\beta = |\beta_s - \beta_r| = (1/2\gamma_z^2)|k_r-k_s|/k_r$. Given that typically $k_s-k_r \sim 2\gamma_z^2\omega_b$ we find,

$$\Delta\gamma \sim \gamma_r(a_w a_r)^{1/2}. \tag{7}$$

It then follows from (5) to (7) that overlapping and transition to chaotic behavior can take place at $a_s \sim a_r$. This crude estimate demonstrates the potential of chaotic behavior for large amplitude sidebands. The above threshold becomes even smaller in case of a multifrequency sideband spectrum.

We will be interested in evaluating the fraction of the phase space that becomes chaotic as a function of the sideband amplitude. This requires the use of a more refined overlapping criterion. Electrons trapped inside the primary island of the main signal still experience perturbations in their motion caused by the sideband. The perturbation is especially felt by these electrons that have the synchrotron frequency ω_b matching the difference between the frequencies of the main signal and the sideband. This condition defines new secondary resonances between the electrons and the sideband. It is the overlapping among the nearby secondary islands, formed inside the primary island, that determines more accurately the break out and extent of the stochastic behavior.

III. PORTRAIT OF THE PHASE SPACE

The electrons are injected into a FEL with energies near the resonant energy γ_r for the main signal ω_r. Expression (3) can be linearized for small excursions $\delta\gamma/\gamma_r << 1$ for electrons not too far from the separatrix. Introducing $\tilde{\gamma} = \gamma - \gamma_r$ and $\psi = (k_w + k_r)z - \omega_r t$ as a new pair of canonical variables and approximating the time $t(z) \simeq z/c\beta_r$ we obtain,

$$H(\tilde{\gamma},\psi;z) = \frac{k_w}{\gamma_r}\tilde{\gamma}^2 + \frac{a_w a_r}{\gamma_r}(\cos\psi + \psi\sin\psi_r) + \frac{a_w a_s}{\gamma_r}\cos(\psi - \delta_s z). \tag{8}$$

In (8) the phase flow is parametrized by the traveled length z inside the wiggler rather than the time t. It was also assumed that the wiggler parameters change slowly compared to the wiggler wavelength $2\pi/k_w$. The term $\sin\psi_r$ parametrizes the rate of change for the resonant energy caused by the change in the wiggler wavelength,

$$\frac{d}{dz}\gamma_r = \frac{k_r a_w a_r}{\gamma_r}\sin\psi_r , \tag{9}$$

where $\psi_r = \pi$ corresponds to an untapered wiggler. The term δ_s in the sideband phase is the Doppler downshifted difference between the signal and the sideband wave numbers,

$$\delta_s = \frac{k_w}{k_r}(k_s - k_r) . \tag{10}$$

In the absence of sidebands, $a_s = 0$, the Hamiltonian H_o is integrable. The unperturbed trajectories in the ponderomotive well are given by $H_o(\tilde{\gamma},\psi) = K$ where K is the reduced energy in the ponderomotive frame. These orbits take the simplest possible form expressed in terms of the action-angle variables (J, θ), defined as,

$$J = \frac{1}{2\pi}\oint d\psi\, \tilde{\gamma}(K,\psi), \qquad \theta = \frac{\partial}{\partial J}\int^{\psi} d\psi'\, \tilde{\gamma}(K,\psi'), \tag{11}$$

where $K = H_o(J)$ and the path of integration is over the unperturbed orbits. For trapped particles in closed trajectories, the action J is related to the area in phase space enclosed by the orbit. For untrapped particles in open trajectories, the path of integration in Eq. (11) depends on the wiggler type. In case of an untapered wiggler, the orbits are periodic and the limits of ψ integration are from 0 to 2π. In case of a tapered wiggler the path of integration is the segment of the trajectory that begins and ends at $\psi = \psi_s$, enclosing the separatrix. Thus, J remains finite, avoiding an infinite jump in action across the separatrix that would result by considering the full orbit length for unbound orbits[18]. J is always periodic in ψ, $J(\tilde{\gamma},\psi_1) = J(\tilde{\gamma},\psi_2)$ for $\psi_1 = \psi_2 + 2\pi$, even when H_o is not (case of tapered wiggler).

Hamiltonian (8) is now transformed under the canonical transformation defined by Eq. (11) into,

$$H(J,\theta;z) = H_o(J) + \frac{a_w a_s}{\gamma_r}\sum_{n=0}^{\infty} Q_n^+(J)\cos(n\theta + \delta_s z) + Q_n^-(J)\cos(n\theta - \delta_s z). \tag{12}$$

$Q_n^{\pm}(J)$ are the Fourier coefficients obtained by the decomposition of the perturbing sideband phase $\psi(J,\theta) - \delta_s z$ into harmonics of the angle θ,

where $\psi(J, \theta)$ is obtained by inverting Eq. (11). In case of constant parameter wiggler J, θ and Q_n (J) are expressed in closed forms given in Appendix A.

$H_o(J)$ is independent of θ so the unperturbed orbits in (J, θ) space are straight lines,

$$J = \text{const.}, \qquad \theta = \theta_o + \kappa_b(J)z \ .$$

The synchrotron wave number $\kappa_b(J)$ is connected to the bounce length L_b and the synchrotron frequency in the laboratory frame $\omega_b(J)$ with the relation,

$$\kappa_b(J) = \frac{dH_o(J)}{dJ} = \frac{2\pi}{L_b(J)} = \frac{\omega_b(J)}{c\beta_z} \ . \tag{13}$$

Since $c = 1$ in the normalized units and $\beta_z = 1$ in the cases of interest, we may use $\omega_b(J)$ in place of $\kappa_b(J)$ as well.

Expression (12) for the transformed Hamiltonian reveals the new resonances emerging when a sideband is turned on. Defining the phase of the nth sideband induced harmonic $\theta^{(n)} = n\theta \pm \delta_s z$, the stationary phase condition reads,

$$\pm\, n\kappa_b(J) - \delta_s = 0, \qquad \text{or} \qquad \pm n\beta_z\omega_b(J) - \delta_s = 0. \tag{14}$$

Thus, particles, originally in unperturbed orbits $J = J_n$, resonate with the sideband when the nth harmonic of their synchrotron period $\omega_b(J_n)$ matches the downshifted frequency difference between the sideband and the carrier signal.

For a given n and sufficiently small a_s we may keep only the resonant term $\theta^{(n)}$ to examine the motion in the vicinity of J_n. This is formally achieved by the canonical transformation,

$$\begin{aligned} \Theta &= n\theta - \delta_s z \;, & I &= \frac{1}{n} J \;, \\ Z &= \delta_s z \;, & I_Z &= \frac{1}{\delta_s} K + \frac{1}{n} J \;, \end{aligned} \tag{15}$$

coming from the generating function $F(\theta,z,I,I_Z) = (n\theta - z)I - zI_Z$. The resulting Hamiltonian is,

$$H_n(I,I_Z,\Theta,Z) = H_o(nI) + \delta_s(I_Z - I) + \frac{a_w a_s}{\gamma_r} Q_n(nI) \cos\Theta + O(a_s^{\,2}) \;. \tag{16}$$

The fixed points (J_n, Θ_n) are found from,

$$\frac{d\Theta}{dZ} = \frac{\partial H_n}{\partial I} = \frac{\partial H_o}{\partial I} - \delta_s = 0, \tag{17a}$$

$$\frac{dI}{dZ} = - \frac{\partial H_n}{\partial \Theta} = \frac{a_w a_s}{\gamma_r} Q_n(nI) \sin\Theta = 0. \tag{17b}$$

Using relations (15) for the transformed variables we recover from (17a) the resonant condition (14) while (17b) indicates $\Theta_n = k\pi / n$, k = 0,1,...,n-1.

In short, a single frequency sideband causes chains of secondary islands to appear inside the original primary island. Each chain corresponds to a given harmonic n and is centered around the stable fixed points J_n, Θ_n. The structure of the phase space is shown in Figs. 1 and 2. They are surfaces of section, created by numerically

integrating the original equations of motion from Hamiltonian Eq. (8) and then recording the intersection point of each trajectory with the plane $z = 2\pi/\delta_s$. The $\tilde{\gamma}$ vs. ψ plots are on the left side in Figs. 1 and 2. The plots on the right side show the same surfaces of section in action-angle variables, produced by the transformations (11). The bounce frequency around a given secondary island is found by linearly expanding the resonant Hamiltonian (16) in $\delta I = I - I_n$. From the resulting pendulum equation and from relations (15) one finds that the secondary synchrotron period Ω_n near the center is given by,

$$\Omega_n = n \left[\left(\frac{d\omega_b}{dJ}\right)_{J_n} \frac{a_w a_s}{\gamma_r} Q_n^{\pm}(J\ ) \right]^{1/2}, \tag{18a}$$

while the half-width of the island δJ_n is

$$\delta J_n = \left[2\, \frac{a_w a_s}{\gamma}\, \frac{Q_n(J_n)}{(d\omega_b/dJ)_{J_n}} \right]^{1/2}. \tag{18b}$$

Representation (12) for the Hamiltonian (8) is formally independent on the details of the transformations (11). Consequently, the same stability analysis applies for constant as well as variable parameter wigglers.

IV. THRESHOLD FOR ERRATIC MOTION

When the sideband amplitude exceeds a certain amplitude α_s regarded as the stochasticity threshold, the presence of even one sideband frequency suffices to transform the regular coherent motion, such as in Fig. 1, to the irregular unbounded motion shown in Fig. 2. The mechanism for this radical change in behavior can be briefly described as follows. The trajectories emanating from the unstable fixed points (X-points) of a secondary island do not actually join smoothly around that island. They intersect infinite times with each other[12,13] due to the effect of the other harmonics $n' \neq n$ that were ignored during the local approximation Eq. (16). A thin layer of fuzzy motion thus surrounds each island chain of given n. As the amplitude a_s increases, the width of each island increases according to (18b) and so does the thickness of the stochastic layer around that island. At a given point the stochastic layers around the two neighboring island chains n and n+1 overlap[14], allowing particles to hop from one island to another. This signifies the beginning of unbounded, random motion in J characterized as stochastic diffusion.

Various methods of different accuracy have been developed for estimating the stochasticity threshold[12,14]. An approximate criterion that works well in most cases is,

$$\delta J_n + \delta J_{n+1} \geq \frac{2}{3} \Delta J_n , \qquad (19)$$

where δJ_n, δJ_{n+1} are the separatrix half-widths and $\Delta J_n = J_{n+1} - J_n$ is the distance between the separatrix centers for the n and n+1 harmonics respectively. For small widths δJ and distances ΔJ compared to J we may expand

$$\omega_b(J_{n+1}) - \omega_b(J_n) = \left(\frac{\partial\omega_b}{\partial J}\right)_{J_n} \Delta J_n \simeq \frac{\omega_b(J_n)}{n} , \tag{20}$$

and use (18b) with $J_{n+1} \simeq J_n$ to obtain the amplitude $a_s(n)$ for overlapping

$$a_s(n) \simeq \left(\frac{2}{3n}\right)^2 \frac{\omega_b(J_n)}{(d\omega_b/dJ)_{J_n}} \left(\frac{\gamma_r}{2a_w Q_n(J_n)}\right) . \tag{21}$$

The outermost islands centered at $J_n \simeq J_s$, correspond to larger shear $d\omega_b(J)/dJ$, smaller $\omega_b(J)$ and higher harmonics n, for given $\omega_s - \omega_r$. According to (21) the threshold $a_s(n)$ is lower near the separatrix and the outermost secondary islands will be the first to overlap. The overlapping is progressively extending to smaller J_n and lower n as a_s increases. The macroscopic stochastic layer first appears near the original separatrix of the primary island and spreads to the interior of the trapped particle bucket. We take the amplitude when the two innermost harmonics overlap as the threshold for "global" stochastic transition, $\alpha_s \equiv a_s(n_1)$. The lowest possible harmonic n_1 for given frequency ω_s is defined by the resonant condition (14). For J_n small we have both δJ and ΔJ of order J and the approximations that led to (21) are not valid. In this case, the exact expressions for J and $\omega_b(J)$ must be applied inside the criterion (19).

We may obtain the dependence of $a_s(n)$ on the various parameters using Eqs. (A4) and (A7), setting $d/dJ = (d\lambda/dJ)\, d/d\lambda$ and utilizing the properties of the elliptic integrals to compute the derivatives in Eq. (21). We find that,

$$\frac{a_s(n)}{a_r} \simeq \frac{1}{n^2} F^2(\lambda_n) , \tag{22}$$

where

$$F(\lambda_n) = \left\{ \frac{E_1(\lambda_n)\ \lambda_n^{\ 2}\ (\lambda_n^{\ 2} - 1)}{E_2(\lambda_n) - (1 - \lambda_n^{\ 2})\ E_1(\lambda_n)} \right\}^{1/2} \left(2Q_n(\lambda_n) \right)^{-1/2} .$$

The threshold for extensive stochasticity $\alpha_s \equiv a_s(n_1)$ is independent of γ and a_w. The trapping parameter λ_n is determined uniquely from J_n according to $\lambda_n = H_o(J_n)\gamma_r/a_w a_r + 1/2$ (see Appendix A). Thus, the sideband frequency ω_s, related to $\omega_b(J_n)$ through the resonant condition (14), is the only parameter that α_s/a_r depends on. The scaling in (22) is still valid in case of large secondary island width with a modification in the numerical factor F.

In Fig. 3a we plot in solid line the threshold α_s for extensive stochasticity, when the two innermost secondary island chains overlap, as a function of the frequency difference $\omega_s - \omega_r$. The dotted line shows the threshold for overlapping between the next two secondary island chains. Some deeply trapped orbits, near the center of the original primary island, still persist when a_s is close to α_s. The extent of the area unaffected by the irregular motion when $a_s \simeq \alpha_s$ is given approximately by $J < J_c$ where $J_c \simeq J_n - \delta J_n$. In Fig. 3b we plot the portion J_c/J_s of the remaining "good" trajectories when the sideband amplitude equals α_s as a function of $\omega_s - \omega_r$. It is seen that the threshold α_s is larger and the extent of the stochastic regime is maximized as well for frequency mismatch near a harmonic of the synchrotron frequency $\omega_b(0)$ at the bottom of the ponderomotive well.

The threshold α_s for overlapping is considerably lower but the extent of the stochastic regime also diminishes for frequencies far from a harmonic of the central synchrotron frequency.

A typical phase portrait for a sideband amplitude a_s slightly above α_s is shown in Fig. 2b. Two different kinds of regions coexist: a stochastic regime where diffusive behavior prevails, interrupted here and there by islands of regular motion, remnants of the original regular motion. The stochastic regimes are interconnected allowing unbounded particle transport. The rate of diffusion as well as the decorrelation times are not uniform in phase space but depend on both J and θ.

When a_s is increased well above α_s the chaotic motion engulfs almost 100% of the phase space (Fig. 2c). The decorrelation time is short everywhere in phase space. In this parameter regime the behavior of the system can be described by a diffusion coefficient D(J) depending on the action J only and insensitive to the frequency of the driving sideband. Total stochastization of the island interior occurs roughly when the sideband amplitude grows to the point where the stable fixed point $\tilde{\gamma} = 0$, $\psi = \pi + \psi_r$ at the center of the original island becomes unstable.

V. NARROW FREQUENCY BAND DIFFUSION

We will examine first the diffusion caused by the presence of one single frequency, large amplitude sideband. This is a relevant approximation in case of a narrow sideband spectrum. The term narrow implies a spectral width $D\omega_s$ much smaller than the frequency separation $\omega_s - \omega_r$, typically of the order $2\gamma_z{}^2\omega_b$. We examine the evolution of a monoenergetic distribution $f(J;z=0) = \delta(J - J_o)$ by numerically integrating the equations of motion. We plot $\langle\Delta J^2\rangle$, $\langle J\rangle$ and $2\langle\Delta J^2\rangle/z$ against the distance z in Figs. 4a, 4b, and 4c respectively. The electrons are initially uniformly distributed in θ with constant action $J_o = 0.7\ J_s$. Different curves in the same frame correspond to different sideband amplitudes a_s at a given frequency ω_s.

For a constant diffusion coefficient **D**, independent of J, the average $\langle J\rangle$ and the mean square deviation $\langle\Delta J^2\rangle = \langle J^2\rangle - \langle J\rangle^2$ would evolve as $\langle\Delta J^2\rangle = (1/2)\ D\ z$, $\langle J\rangle = J_o$. The dashed curves in Figs. 4a - 4c correspond to a sideband amplitude a_s below the stochasticity threshold α_s. The deviation $\langle\Delta J^2\rangle$ asymtotes to a constant after an initial increase while the ratio $\langle\Delta J^2\rangle/z$ tends to zero for large z. In this case stochasticity is localized. Different stochastic regimes are still separated by "good" integrable orbits (KAM surfaces) located in between. Electrons diffuse until they are stopped at the boundaries of the stochastic regimes that "compartmentalize" the phase space. The solid curves in Figs. 4a - 4c correspond to sideband amplitude above the global stochasticity threshold α_s. This means that the last good orbit has been destroyed allowing different stochastic regimes to interconnect. $\langle\Delta J^2\rangle$ now increases monotonically and the ratio $\langle\Delta J^2\rangle/z$ remains finite for large z. The fact that the diffusion rate is not

constant, and that the average $\langle J \rangle$ changes away from the initial value J_o, shows that D depends strongly on J.

In principle, one could determine a local D(J) by advancing test distributions $\delta(J-J_o)$ of various J_o over short distances z. Then the Fokker-Planck equation for any initial distribution $f_o(J)$ could be solved numerically using D(J). Here, instead, we elect to measure directly the effective diffusion rate associated with a given type of initial distribution. We do so by integrating numerically the equations of motion, Hamiltonian (8), for a number of particles (typically 400) assuming constant amplitude for the electromagnetic fields. A uniform initial distribution in phase space with trapped particles inside the (unperturbed) separatrix is chosen, $f_o(J) = [1 - S(J - J_s)]/J_s$ where S is the step function. This situation is relevant with the operation of tapered wiggler FELs where the trapped particles in the ponderomotive bucket are decelerating, falling quickly behind the untrapped particles and thus creating large distribution gradients near the separatrix.

The two questions of practical interest are (a) what percentage of the particles will eventually get detrapped and (b) how fast do they leak outside the separatrix. For our uniform initial distribution the maximum fraction of particles becoming detrapped equals the fraction of the inside of the separatrix area that becomes chaotic. In Fig. 5a we plot the fraction f_d of the particles that cross the original separatrix J_s as a function of the traveled wiggler length for values of $q = a_s/a_r$ below the threshold for extended stochasticity. In all cases an initial stage of quick diffusion is followed by a long period where the average number of untrapped particles remains practically constant. The results are consistent with the existence of a boundary in phase space (KAM surface) separating two regimes: the one of unbounded, chaotic motion

from the one filled with regular, coherent orbits of particles that remain trapped. Only electrons in the area between the last integrable surface and the old unperturbed separatrix will diffuse until that area is depleted. A fraction 1 - f_d of the original primary island area will remain trapped for an arbitrarily long time, as long as a_s remains below the threshold α_s associated with the particular sideband frequency. This fraction is shrinking as a_s increases and the bucket "peels off". The situation when a_s exceeds α_s is shown in Fig. 5b. The fraction of untrapped particles f_d reaches 1 in all cases, meaning complete absence of particle confinement in the bucket. All particles can eventually escape with a rate that increases with increasing a_s.

Numerical results showing the fraction of detrapped particles f_d after 100 wiggler periods as a function of a_s/a_r are plotted in Fig. 6 for various sideband frequencies $|\omega_s - \omega_r|$. The length over which approximately half of the initially trapped particles get detrapped will be discussed in the next section, in comparison with the diffusion rates from other types of sideband spectra.

VI. BROAD FREQUENCY BAND DIFFUSION

So far stochastic electron detrapping caused by a single frequency sideband has been examined. It was argued that when the excited sideband spectrum is narrow enough, i.e., $D\omega_s \ll 2\gamma_z^2\omega_b$, the situation can be reasonably approximated by a single frequency sideband. Here, we consider the situation when a broad spectrum of frequencies have been excited, $D\omega_s \geq 2\gamma_z^2\omega_b$. We will make a distinction between a continuous and a discrete spectrum. In case of a discrete spectrum the distance between two nearby sideband frequencies is much larger than the width of an individual spectral line. In the opposite case, when various peaks in the spectrum merge together, we will talk about a continuous spectrum. We may model numerically both cases by introducing a modulation in the sideband phase of Hamiltonian Eq. (11),

$$\tilde{H}(\gamma,\psi;z) = \frac{k_w}{\gamma_r}\tilde{\gamma}^2 + \frac{a_w a_r}{\gamma_r}(\cos\psi + \psi\sin\psi_r) + \frac{a_w a_s}{\gamma_r}\cos(\psi + A\sin\nu z - \delta_s z)\ , \tag{23}$$

that is transformed in action-angle variables as

$$H(J,\theta;z) = H_o(J) + H_1(J,\theta;z),$$

$$H_1(J,\theta;z) = \tag{24}$$

$$\frac{a_w a_s}{\gamma_r}\sum_{m=-\infty}^{\infty} J_m(A)\sum_{n=0}^{\infty} Q_n^+(J)\cos[n\theta + \delta_s(m)z] + Q_n^-(J)\cos[n\theta - \delta_s(m)z].$$

The frequency mismatch values $\delta_s(m)$ and the corresponding sideband frequencies $\omega_s(m)$ are given by,

$$\delta_s(m) = \delta_o + m\,\nu, \qquad \omega_s(m) = \omega_{so} + 2\,m\,\gamma_z^{\,2}\nu, \tag{25}$$

where $\omega_{so} = \omega_r + 2\gamma_z^{\,2}\delta_o$. Since the Bessel function coefficients become vanishingly small, $J_m(A) \ll 1$ for $A > m$, the width of the spectrum is given by $D\delta_s \sim A\,\nu$ or $D\omega_s \simeq 2\gamma_z^{\,2}A\nu$.

In order to examine the connection between diffusion rates and the types of the sideband spectra, we divide the latter into three general categories: narrow, broad discrete and broad continuous. The passage from one regime to the other is not gradual but characterized by abrupt changes in the diffusion coefficients. Thus, from the diffusion point of view, the distinction among the spectral types is not arbitrary but based on certain relations between the parameters A and ν. In all three regimes of the parameter space the rate of diffusion is proportional to the ratio of the total sideband power to the carrier power. The scaling of the coefficients of this proportionality on the various FEL parameters, however, differs from one regime to the other.

Both cases of the broad spectrum are characterized by a width $D\omega_s$ in the excited frequencies that is larger than the upshifted synchrotron frequency ω_b,

$$D\omega_s > 2\,\gamma_z^{\,2}\omega_b \quad \text{, equivalent to} \quad \nu > \frac{\omega_b}{A} \quad , \tag{26}$$

with $A \gg 1$. The further distinction between discrete or continuous spectrum is related to the separation between nearby frequencies. We find that when $\omega_b/A^{1/2} > \nu > \omega_b/A$ the diffusion rate agrees well with the quasilinear diffusion coefficient. A different coefficient is derived for the case when $\nu > \omega_b/A^{1/2} > \omega_b/A$, in agreement with the numerical simulations. Consequenlty the separation $\omega_b/A^{1/2}$ between

nearby modes marks the transition from a discrete to a continuous type of behavior. Departure from the quasilinear diffusion coefficient has also been observed numerically in previous literature[19] when the frequency separation between nearby modes was not "small enough". Here a condition for the discrete-to-continuous transition has beeen obtained.

For any spectral type, the sideband amplitude must be above the stochastisity threshold in order to trigger electron diffusion. Using the same method of nearby resonance overlapping as in Sec. III, and the Hamiltonian Eq. (25) we find that the threshold in case of a multifrequency spectrum is given by,

$$\tilde{\alpha}_s \simeq \frac{\nu}{\omega_b J_m(A)} \alpha_s , \qquad (27)$$

where α_s is the threshold for the single sideband frequency. Thus $\tilde{\alpha}_s$ decreases with decreasing frequency separation ν. Condition (27) guarantees the stochastization of the particle orbits. The frequency separation among sidebands must be limited by the additional condition $\nu < \omega_b/A^{1/2}$, as stated in the previous paragraph, if one wishes to simulate quasilinear diffusion with a discrete spectrum.

A. Broad Discrete Spectrum.

We now evaluate the diffusion coefficient for a broad, discrete spectrum. The equation of motion for J can be written as,

$$\frac{dJ}{dz} = - \frac{\partial H_1}{\partial \theta} = - \frac{\partial H_1}{\partial \psi} \frac{d\psi}{d\theta} = \Lambda \frac{a_w a_s}{\gamma_r} \sin \left(\psi + A \sin \nu z - \delta_s z\right) \left|V(\psi_{mx}) - V(\psi)\right|^{1/2},$$

using Eq. (A2) for $d\psi/d\theta$. Due to the presence of many frequencies in the spectrum $J(z)$ executes a complicated oscillatory motion with the average $\langle J \rangle$ changing very little most of the time. J however receives a large kick ΔJ near resonances, where the phase $\Phi = \psi + A \sin \nu z - \delta_s z$ of the multifrequency perturbation H_1 varies slowly. The resonant condition is,

$$\frac{d\Phi}{dz} = \frac{k_w \tilde{\gamma}}{\gamma_r} + A\, \nu \sin \nu z - \delta_s = 0, \tag{28}$$

at some $z = z_i$. Given that $k_w \tilde{\gamma}/\gamma_r \leq \omega_b$, collective effects due to many frequencies are important for the resonance in Eq. (28) when $A\, \nu > \omega_b$. On this basis inequality (27) signifies the transition from a narrow to a broad spectrum. Let us consider the case $A\, \nu >> \omega_b$. Then the resonances occur at $z_i \simeq i\pi/\nu$, i integer, and the interval between successive resonances is $\Delta z \simeq \pi/\nu$. Expanding the phase $\Phi(z)$ in the equation of motion for J around the resonance z_i,

$$\Phi(z) \simeq \Phi_i + \frac{1}{2}\left[\omega_b^{\,2}(\sin\psi_i + \sin\psi_r) + A\, \nu^2 \cos \nu z_i\right](z - z_i)^2, \tag{29}$$

and extending the limits of integration to $z = \pm\infty$ we obtain,

$$DJ_i \simeq \frac{a_w a_r}{\gamma_r}\left(\frac{2\pi}{\omega_b^{\,2}}\right)^{1/2} \frac{\cos\left(\Phi_n \pm \frac{\pi}{4}\right)\ \left|V(\psi_{mx}) - V(\psi_i)\right|^{1/2}}{\left((\sin\psi_i + \sin\psi_r) + A\left(\frac{\nu}{\omega_b}\right)^2\right)^{1/2}}, \tag{30}$$

where $\psi_{mx}(J)$ is the turning point for an unperturbed trajectory of given J. When $A\, \nu^2 >> \omega_b^{\,2}$ both Δz and ΔJ depend on the features ν and A of the sideband spectrum and not on the bounce frequency ω_b. We classify the cases with frequency separation $\nu > \omega_b/A^{1/2}$ as broad discrete spectra. They obey a distinct scaling in the diffusion coefficient that will be

derived below. We find the cases with $\nu < \omega_b/A^{1/2}$ to agree numerically with the quasilinear diffusion that will be studied in the next subsection.

The resonant phases Φ_i between two successive jumps of ΔJ become quickly decorrelated when a_s grows above the stochastisity threshold. Because the relation between Φ_i and ψ_i involves the distance z_i, Φ_i and ψ_i will also become decorrelated, $\langle \cos\Phi_i \ \cos\psi_i \rangle = 0$. Assuming complete decorrelation between two successive jumps we obtain,

$$D_w = \frac{2 \langle \Delta J_i^2 \rangle}{Dz} = \frac{2 \, a_w^{\,2} a_r^{\,2}}{\gamma_r^{\,2} A \, \nu} \langle \, |V(\psi_{mx}) - V(\psi_i)| \, \rangle, \tag{31}$$

where the angular brackets $\langle \ldots \rangle$ signify the average over ψ_i for constant J. For practical purposes it is more convenient to rescale the diffusion coefficient so that the distance $\underline{z} = z/\lambda_w$ is measured in terms of wiggler wavelengths and the action $\underline{J} = J/J_s$ signifies the location relative to the separatrix. In these units, using Eq. (A4) for J_s and setting $\nu / \omega_b(0) = r$ we obtain,

$$\underline{D}_w = \frac{2\pi}{k_w} \frac{D_w}{J_s^{\,2}} \sim \frac{\pi^3}{8} \frac{g^2 \zeta}{A \ r} \left(\frac{a_w a_r}{1 + a_w^{\,2}} \right)^{1/2} \frac{a_s^{\,2}}{a_r^{\,2}} \, . \tag{32}$$

The term g is a scaling factor, the ratio of the separatrix area for an untapered wiggler to that of a tapered wiggler, $g = J_s(\psi_r = \pi)/J(\psi_r)$, and depends only on ψ_r[1]. The term $\zeta \sim 1$, coming from the averaging over ψ_i in Eq. (31), is computed in Appendix B. The typical diffusion length L_d, the traveled distance inside the wiggler over which the average trapped particle crosses the separatrix, is estimated from the diffusion Eq.

(22) by taking $\langle \Delta J^2 \rangle = J_s^{\,2}$, $L_d \sim J_s^{\,2}/D_w$. Thus, the diffusion length in wiggler periods $N_d = L_d/\lambda_w$ is the inverse of $\underline{D}_w$,

$$N_d \sim \frac{1}{\underline{D}_w}. \tag{33}$$

B. Broad Continuous Spectrum

Next we consider the case of a sideband wave package,

$$a_s(z,t) = \frac{1}{2\pi} \int dk_s a_s(k_s)\, e^{ik_s z - i\omega(k_s)t} , \tag{34}$$

of finite spectral width Dk_s centered around k_{so}. Our purpose is to obtain the diffusion coefficient for a continuous spectrum using the methods of the quasilinear theory. Upon using exression (34) for the fields, the Hamiltonian representation in action-angle variables assumes the form,

$$H(J,\theta;z) = H_o(J) + \frac{a_w}{\gamma_r} \sum_{n=0}^{\infty} Q_n^{\pm}(J) \int dk_s a_s(k_s) \cos\,[n\theta \pm \delta(k_s)z] , \tag{35}$$

where $\delta(k_s)$ is given by,

$$\delta(k_s) = \delta_{so} + (k_s - k_{so}) \frac{k_w}{k_r} + \left(\frac{v(k_s) - v(k_r)}{c}\right) k_{so} , \tag{36}$$

and $\delta_{so} = (k_w/k_r)(k_{so} - k_r)$ in the spirit of Eq. (10). The last parenthesis in the right-hand side of (36) is of order $(\omega_p/\omega_r)^2$ resulting from the dispersive effects in the sideband spectrum. The

finite k_w provides phase slippage among the ponderomotive phases of various wavenumbers k_s, a necessary condition for the validity of the quasilinear theory, even when the dispersive effects of the last term in (36) are negligible, i.e., $\gamma_z^2(\omega_p/\omega_r)^2 << Dk_s/k_s$.

The resonant condition between a sideband wavelength k_s and a given harmonic n now reads $\pm n\omega_b(J_n) = \delta(k_s)$. For each harmonic n there exists a wide band of resonant orbits centered around J_{no} and of width DJ_n defined by,

$$DJ_n = (d\Omega_b/dJ)_{Jno}^{-1} (k_w/k_r)\, Dk_s, \quad \pm n\omega_b(J_{no}) = \delta_{so}. \tag{37}$$

One condition for the applicability of the quasilinear theory is that the phase mixing due to Dk_s occurs much faster than the bounce period around a secondary island in phase space. In this way, electrons, that otherwise would execute periodic orbits around some fixed point, lose coherence sufficiently fast to allow random motion of the Fokker-Planck type. Taking the decorrelation length for the phase $l_d \sim 2\pi/D\delta(k_s)$ and applying Eq. (18a) for the bounce period around the nth harmonic we obtain

$$\frac{Dk_s}{k_s} >> \frac{n}{2}\left(\frac{\pi Q_n^{\pm}}{\sqrt{2}}\right)^{1/2}\left(\frac{\Lambda a_w a_r}{1 + a_w^2}\right)^{1/2}. \tag{38}$$

Inequality (38) guarantees the diffusive behavior within the band DJ_n around J_{no} given by (37). Large scale diffusion, permitting transport of deeply trapped particles across the separatrix J_s of the original primary island, requires that different stochastic bands touch each other, $DJ_n + DJ_{n+1} \geq \Delta J_n$, or, using a similar approach as in Eqs. (19) and (20),

$$\frac{Dk_s}{k_s} > \frac{1}{8n}\left(\frac{\pi Q_n^{\pm}}{\sqrt{2}}\right)^{1/2}\left(\frac{\Lambda a_w a_r}{1 + a_w^2}\right)^{1/2} . \tag{39}$$

The right-hand sides of Eqs. (38) and (39) are of the same order as the upshifted synchrotron period for the main bucket. Thus both conditions are satisfied when,

$$\frac{Dk_s}{k_s} >> 2\, \gamma_z^2\, \frac{\kappa_b(0)}{k_r} . \tag{40}$$

Note that (40) is the same as the condition (26) that defines the wide spectrum, obtained in the previous subsection using different arguments. Then the evolution of the initial distribution $f_o(J)$ is globally described by a diffusion equation,

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial J} D_q(J) \frac{\partial f}{\partial J} . \tag{41}$$

Applying the standard procedures of the quasilinear theory[20,21] (see appendix C) and taking the limit of small growth rate for the sidebands, $Im(k_s)/k_s << 1$, we obtain,

$$\begin{aligned} D_q(J) &= \frac{k_r}{4} \frac{k_r}{k_w} \frac{a_w^2}{\gamma_r^2} \sum_{n=0}^{\infty} n^2\, |Q_n^{\pm}(J)|^2 \int dk_s W_s(k_s)\, \delta(k_s - k_n), \\ k_n &= k_r \pm \frac{k_r}{k_w}\, n\, \kappa_b(J) . \end{aligned} \tag{42}$$

According to the condition (40) for the validity of the quasilinear

theory, the wave package has a wide spectrum $Dk_s \sim N(k_r/k_w)\omega_b(0)$ with N large. The wave components $a_s(k_n)$ fall off slowly for $n < N$ while the Fourier coefficients Q_n (J) decay rapidly with n. Then we may factor out the average spectral power density $W_s = (1/Dk_s)\int dk_s\, W_s(k_s) \simeq a_s^2/Dk_s$ in Eq. (34) getting,

$$D_q(J) \simeq \frac{k_r}{4}\frac{k_r}{k_w}\frac{a_w^2}{\gamma_r^2} W_s \sum_{n=0}^{\infty} n^2 |Q_n^{\pm}(J)|^2. \qquad (43)$$

The infinite sum in the right-hand side of (43) is computed in Appendix C. The summation technique does not require the knowledge of the individual coefficients $Q_n(J)$ and the result depends only on the quantities J and $\omega_b(J)$ for the unperturbed Hamiltonian $H_o(J)$. We then obtain the diffusion coefficient in closed form,

$$D_q(J) = \frac{k_r\, a_w^2\, \gamma_z^2\, W_s\, k_w\, J}{\gamma_r^3 \omega_b(J)} \quad . \qquad (44)$$

We note in passing that the method used to obtain expression (44) for $D_q(J)$ is quite general and valid for any integrable dynamical system $H_o(J)$ that is subject to an external perturbation. In particular, it should be applicable to a variety of RF heating methods in fusion plasmas, commonly involving a strong, narrow-band pump wave embedded in a wide, parametrically excited, fluctuation spectrum.

Using the expressions (A3) and (A7) for the action J and the synchrotron frequency $\omega_b(J)$ we find that the diffusion coefficient goes to zero at the centre of the primary island $J = 0$, has a logarithmic singularity at the separatrix $J = J_s$ and falls off away from it. In

normalized units, with the wiggler wavelength λ_w as the unit length and the action J_s at the separatrix as the unit action, we have,

$$\underline{D}_q(J) = \frac{2\pi}{k_w} \frac{D_q(J)}{J_s^2} . \tag{45}$$

Choosing the value $J = J_s/2$ inside $D_q(J)$ we obtain an estimate for the effective diffusion coefficient associated with the uniformly filled distribution,

$$\underline{D}_q \sim \frac{\pi}{4} \frac{g\, a_w a_r}{\Lambda\, (1+a_w^2)} \frac{a_s^2}{Dk_s\, a_r^2} , \tag{46}$$

where g is the same scaling factor as in Eq. (32).

Note that both expressions (32) and (46), corresponding to the two different spectral types, are independent of γ_r. Thus, for the same wiggler parameters and total sideband power, the detrapping distance in wiggler periods is independent of the electron beam energy. The dependence of the effective diffusion rate on the wiggler tapering enters through the form factor $g(\psi_r) = J_s(\pi)/J(\psi_r)$. As the rate of tapering increases and ψ_r shifts further from π, g inreases[1] and accelerates the effective diffusion rate. This happens because the trapped area in phase space, parametrized by J_s, is shrinking as the tapering progresses, while the sideband induced excursions remain the same, depending mainly on the sideband strength and spectrum. This

shortens the average detrapping time for a particle. The diffusion by broad discrete spectrum, Eq. (32) scales as g^2, while the quasilinear diffusion, Eq. (46), scales as g. Thus the former is affected more by tapering than the latter.

VII. NUMERICAL RESULTS

The numerically computed diffusion coefficient $\underline{D}$ and the diffusion length in wiggler periods $N_d = 1/\underline{D}$ are plotted against the sideband to carrier power ratio $P = \sum_n a_s^2(\omega_n)/a_r^2 = W_s/W_r$ in Figs. 7 and 8 respectively for the three different types of spectra. We have integrated numerically the equations of motion for 400 particles of a uniform initial distribution inside the bucket. The field intensities remained constant at $a_r = 5\times10^{-5}$, $a_w = 2$ and $\gamma_r = 25$. All the numerical results in this paper correspond to a tapered wiggler with $\psi_r = 7\pi/6$. A clear separation in the diffusion rates is observed among the various spectral types. The narrow frequency results (triangles) were obtained using the Hamiltonian (8) with a single sideband frequency $\omega_s/\omega_r =$ 1.016. The results for a broad discrete spectrum (circles) were obtained using (23) with $A = 20$, $\omega_s/\omega_r = 1.016$ and $\nu = 0.5\ \delta_s$. The continuous spectrum (squares) was modeled by $A = 100$, $\nu = 0.05\ \delta_s$. The solid lines, corresponding to the theoretical results of Eqs. (32) and (46), are in good agreement with the numerics. Theoretical predictions for the single frequency case were not made. We stress, however, the difference between single frequency results and quasilinear theory in this case. The agreement that has been observed in some other cases[22] is not generic but particular to certain systems.

Figure 9 illustrates the difference of the electron response to different sideband spectra. The plots on the left side are typical orbits $J(z)$ for selected particles along the wiggler. The trajectories in all plots are generated by the same initial conditions for the electrons and the same FEL parameters a_w, a_r and k_w, as well as the same mean square sideband power $\langle a_s^2\rangle$. The spectral parameters A and ν,

however, are different so that each of the figures (a) to (c) corresponds to one of the three spectral types defined earlier. The dashed line marks the position of the unperturbed separatrix J_s. The corresponding distribution functions $f(J,z)$ at the beginning, $z = 0$, halfway inside, $z = 50\lambda_w$, and at the end, $z = 100\lambda_w$ of the wiggler are plotted in the right-hand side of Figs. 9a-9c respectively.

In Fig. 10 we plot the diffusion coefficient for a uniformly filled bucket as a function of the energy γ_r, fixing the wiggler parameters. It is clear that the diffusion rate (measured again in number of wiggler periods) is independent of the beam energy, provided the synchrotron frequency ω_b stays in the same parameter regime.

Once the diffusion coefficients are known, some estimate can be made of the related reduction in efficiency over the wiggler length. The number of trapped particles at any point z is given by

$n_b = \int_0^{J_s} dJ\, f(J,z)$. Using the diffusion Eq. (41) with $D(J=0) = 0$ one obtains the rate of change in the number of trapped particles,

$$\frac{dn_b}{dz} = n_b(z)\, D(J_s)\, \frac{\partial f(J_s,z)}{\partial z} . \qquad (47)$$

The leakage rate for trapped particles changes along z as the slope of the distribution $f(J,z)$ changes. To estimate the average leakage rate we assume that $f(J,z)$ remains Gaussian in J with an average width equal to the separatrix action J_s. We estimate from (47) the e-folding length $L_d = - n_b^{-1}(dn_b/dz)$ for the number n_b of trapped particles,

$$L_d = D(J_s)\, J_s^{-2}. \qquad (48)$$

Assuming that $n_b(z) = n_b(0) \exp(-z/L_d)$, the number of detrapped particles between z and $z + \Delta z$ is $\Delta n(z) = n_b(0) L_d^{-1} \exp(-z/L_d)\, \Delta z$. These particles gave up an amount of energy $\Delta E(z) = [\gamma_r(0) - \gamma_r(z)]\, \Delta n(z)$ as radiation. Integrating $\Delta E(z)$ over the wiggler length for a linearly tapered wiggler $\gamma_r(z) = \gamma_r(0) - z\Delta\gamma/L_w$, we find the total energy extracted from the particles that were detrapped at some point inside the wiggler. Adding the contribution $[\gamma_r(0) - \gamma_r(L_w)]\, n_b(L_w)$ from the particles that remained trapped throughout the wiggler length, we come up with,

$$\eta = \eta_o \frac{L_d}{L_w} \left(1 - \exp\left(- \frac{L_w}{L_d}\right) \right) , \qquad (49)$$

where $\eta_o = \Delta\gamma/\gamma_r(0)$ is the efficiency without induced diffusion. The loss of amplification will, in general, be distributed among all the radiation modes and (49) reflects the total power loss in all frequencies. The extraction efficiency η for a linearly tapered wiggler is plotted in Fig. 11 versus the sideband to carrier power ratio P, obtaining the corresponding value for L_d/L_w from the results in Fig. 8.

VIII. CONCLUSION

The diffusion in phase space caused by sideband excitation during FEL operation was studied. It was shown that the characteristic rates for this process depend on the structure of the sideband spectrum, falling into one of the following general categories: narrow, wide discrete or wide continuous spectrum. In all cases, the diffusion coefficient was found proportional to the ratio of the total power in the sidebands to the power in the main FEL signal $D \propto C\ W_s/W_r$. The coefficient C, however, is connected to the spectral type under consideration. From Eqs. (32) and (46) we see that, apart from numerical factors of order unity, C scales as $(a_w a_r)^l$ with $l = 1/2$ for a discrete and $l = 1$ for a continuous spectrum. Therefore, given the typical FEL values of $a_w \leq 10$ and $a_r \leq 10^{-3}$ an order of magnitude reduction in diffusion occurs in the transition from a discrete to a continuous sideband spectrum. It was also observed numerically that the highest diffusion rate occurs when all the sideband power is in a single frequency. In this case, however, a portion of the particles will remain trapped for arbitrary long wigglers if the sideband amplitude is below the threshold for extensive stochasticity. The stochasticity threshold is progressively reduced as the sideband power is distributed into an increasing number of frequencies. Yet the rate of diffusion also slows down with increasing spectral width and decreasing mode separation. Thus, the minimum reduction in the FEL energy extraction efficiency will occur for continuous sideband spectra. Although control of the sideband structure does not seem plausible, experiments show that a wide spectrum is naturally excited during FEL operation. This would allow enough power build-up before serious deterioration in efficiency, due to detrapping,

to occur. The diffusion length, measured in wiggler periods, is independent of the beam energy γ under the same wiggler parameters, for all the spectral types. Our results have been obtained for radiation fields of constant amplitude. Inclusion of the time evolution for both the carrier and the sidebands will modify the detrapping rates by changing both the diffusion rate as well as the size of the separatrix. This subject is left for future investigation.

ACKNOWLEDGEMENT

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APPENDIX A: TRANSFORMATIONS IN ACTION-ANGLE VARIABLES

The relations between $\tilde{\gamma}$, ψ and the action-angle variables J, θ are given in closed forms in case of an untapered wiggler. Starting from the general expression (11) and using (8) we have,

$$\frac{1}{\pi}\int_{\psi_{mn}}^{\psi_{mx}} d\psi \left(\frac{\gamma_r H_o}{k_w} - V(\psi) \right)^{1/2} , \tag{A1}$$

$$\theta = \frac{\gamma_r \kappa_b(J)}{2k_w} \int_0^{\psi} d\psi' \left(\frac{\gamma_r H_o}{k_w} - \frac{a_w a_r}{k_w} V(\psi) \right)^{-1/2} , \tag{A2}$$

where

$$V(\psi) = \left(\cos\psi + \psi \sin\psi_r \right) .$$

Using $H_s = -(a_w a_s/\gamma_r)\, V(\psi_r)$ we obtain the action at the separatrix,

$$J_s = \left(\frac{a_w a_r}{\gamma_r}\right)^{1/2} \frac{1}{\pi}\int_{\psi_{mn}}^{\psi_{mx}} d\psi \left(V(\psi) - V(\psi_r) \right)^{1/2} .$$

In case of an untapered wiggler $\psi_r = 0$ Eqs. (A1)-(A2) yield,

$$J = \begin{cases} J_s \left(E_2(\lambda) - (1-\lambda^2)E_1(\lambda)\right) , & \lambda^2 < 1 \\ 2J_s\lambda\, E_2\left(\frac{1}{\lambda}\right) , & \lambda^2 > 1 \end{cases} \tag{A3}$$

$$J_s = \frac{8}{\pi}\left(\frac{\Lambda a_w a_r k_r}{4k_w}\right)^{1/2} , \tag{A4}$$

$$\sin\frac{\psi}{2} = \begin{cases} \lambda\ \mathrm{sn}\left(\frac{2}{\pi} E_1(\lambda)\theta\right) , & \lambda^2 < 1, \\ \mathrm{sn}\ \left(\frac{1}{\pi} E_1\left(\frac{1}{\lambda}\right)\theta\right) , & \lambda^2 > 1, \end{cases} \tag{A5}$$

where E_1 and E_2 are the complete elliptic integrals of the first and second kind, sn is the Jacobi elliptic sine function, and

$$\lambda^2 = \frac{\gamma_r H_o}{2a_w a_r} + \frac{1}{2}, \tag{A6}$$

is the trapping parameter ($\lambda^2 < 1$ for trapped particles). Using $\kappa_b(J) = (\partial J/\partial H_o)^{-1}$ and (A1) we determine the bounce (synchrotron) frequency

$$\kappa_b(J) = \begin{cases} \kappa_b(0)\ \frac{\pi}{2E_1(\lambda)} , & \lambda^2 < 1 \\ \kappa_b(0)\ \frac{\pi\lambda}{E_1(1/\lambda)} , & \lambda^2 > 1 \end{cases} \tag{A7}$$

where

$$\kappa_b(0) = \frac{1}{\gamma_r}\left(\frac{\Lambda}{2} a_w a_r k_w k_r\right)^{1/2}, \tag{A8}$$

is the bounce frequency at the bottom of the well. J and λ^2 are mutually related through (A3) - (A6) and they uniquely label the trajectories.

The Fourier coefficients of the expansion (12) can also be expressed in closed form. They are computed by integration in the complex plane around the singularities, utilizing the double periodicity properties of the Jacobi elliptic functions to obtain,

$$Q_n^{\pm} = -(\pm 1)^n \frac{n\pi^2}{E_1^2(\lambda)} \quad \frac{q^{\frac{n}{2}}}{1-(-q)^n}, \ q = \exp\left(\frac{\pi E_1'(\lambda)}{E_1(\lambda)}\right), \ \lambda^2 < 1, \tag{A9}$$

$$Q_n^{\pm} = -\frac{n\pi^2\lambda^2}{E_1^2(1/\lambda)}\, q^n \left(\frac{1}{1-q^{2n}} \pm \frac{1}{1+q^{2n}}\right) , \quad q = \exp\left(\frac{\pi E_1'(1/\lambda)}{E_1(1/\lambda)}\right) , \quad \lambda^2 > 1$$

for $n \neq 0$ and,

$$Q_0^{\pm} = 2\,\frac{E_2(\lambda)}{E_1(\lambda)} - 1 \qquad \lambda^2 < 1 , \qquad Q_0^{\pm} = 1 - 2\lambda^2\left(1 - \frac{E_2(1/\lambda)}{E_1(1/\lambda)}\right) , \quad \lambda^2 < 1,$$

where $E_1'(\lambda^2) \equiv E_1(1 - \lambda^2)$.

APPENDIX B : PHASE AVERAGING OVER CONSTANT J

The phase average $< |V(\psi_{mx}) - V(\psi_i)| >$ over constant J is given, for $\psi_r = 0$, by,

$$<\ldots\ldots> = \frac{1}{2\pi}\int_0^{2\pi} d\theta_i \;\left|\; \cos\psi_{mx}(J) - \cos\psi_i(J,\theta_i) \;\right|$$

$$= \frac{4}{2\pi}\int_0^{\psi_{mx}} d\psi_i \left(\frac{d\theta_i}{d\psi_i}\right) \left|\; \cos\psi_{mx}(J) - \cos\psi_i \;\right| . \qquad \text{(B1)}$$

Substituting $d\theta_n/d\psi_i$ from (A2) and using $\cos\,\psi_{mx} = \gamma_r H(J)/a_w a_r$ one obtains,

$$<\ldots\ldots> = \frac{\gamma_r \kappa_b}{\pi\, a_w a_r}\int_0^{\psi_{mx}} d\psi_i \left(\frac{\gamma_r H}{k_w} - \frac{a_w a_r}{k_w}\cos\,\psi_i\right)^{1/2}$$

$$= \sqrt{2}\left(\frac{E_2(\lambda^2)}{E_1(\lambda^2)} - (\,1 - \lambda^2)\right) , \qquad \text{(B2)}$$

where λ^2 was defined in (A6).

APPENDIX C : COMPUTATION OF THE QUASILINEAR DIFFUSION COEFFICIENT

We consider the evolutic n of the electron distribution in the presence of a sideband wave package,

$$a_s(z,t) = \frac{1}{2\pi} \int dk_s a_s(k_s)\, e^{ik_s z - i\omega(k_s)t} , \tag{C1}$$

of width Dk_s. The interaction Hamiltonian in action-angle variables, derived in Sec. IV.(b) is,

$$H(J,\theta;z) = H_0(J) + \frac{a_w}{\gamma_r} \sum_{m=0}^{\infty} Q_n^{\pm}(J) \int dk_s a_s(k_s) \cos\,[n\theta \pm \delta(k_s)z] \,. \tag{C2}$$

The evolution of the distribution function $f(J,\theta;z)$, under the Hamiltonian flow,

$$\frac{d\theta}{dz} = \frac{\partial H}{\partial J} , \quad \frac{dJ}{dz} = - \frac{\partial H}{\partial \theta} , \tag{C3}$$

is given by,

$$\frac{\partial f}{\partial z} + \frac{d\theta}{dz}\frac{\partial f}{\partial \theta} + \frac{dJ}{dz}\frac{\partial f}{\partial J} = 0 \,. \tag{C4}$$

We separate the distribution $f(J,\theta;z)$ into a slowly varying part $f_0(J;z)$ = $\langle f \rangle$ and a fluctuating part $\delta f(J,\theta;z) = f - \langle f \rangle$. The averaging operator is defined by,

$$\langle f \rangle = \int_0^L \frac{dz}{L} \int_0^{2\pi} \frac{d\theta}{2\pi} f \,.$$

It is implied in the above definition that the characteristic length for $f_o(J;z)$ is longer than the synchrotron length $L = 2\pi/\kappa_b$. We then obtain from (C4),

$$\frac{\partial \tilde{f}}{\partial z} + \frac{d\theta}{dz}\frac{\partial \tilde{f}}{\partial \theta} + \frac{dJ}{dz}\frac{\partial \tilde{f}}{\partial J} = -\frac{dJ}{dz}\frac{\partial f_o}{\partial J} - \frac{\partial}{\partial J}\left[\frac{dJ}{dz}\tilde{f} - \left\langle \frac{dJ}{dz}\tilde{f}\right\rangle\right], \qquad \text{(C5)}$$

$$\frac{\partial}{\partial z} f_o = -\frac{\partial}{\partial J}\left\langle \frac{dJ}{dz}\tilde{f}\right\rangle . \qquad \text{(C6)}$$

Using Eqs. (C2) and (C3) inside Eq. (C5) and ignoring the last bracketed term in the right-hand side we obtain,

$$\tilde{f} = -\frac{a_w}{2\gamma_r}\sum_{n=1}^{\infty}\int dk_s \frac{n\, Q_n^{\pm}(J)\, a_s(k_s)}{n\, \kappa_b(J) \pm \delta(k_s)}\frac{\partial f_o}{\partial J} e^{i[n\theta \pm \delta(k_s)]} + cc . \qquad \text{(C7)}$$

Substituting (C7) in (C6) we have,

$$\frac{\partial}{\partial z} f_o = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_s \int_{-\infty}^{\infty} dq_s \int_0^L \frac{dz}{L}\int_0^{2\pi}\frac{d\theta}{2\pi}\frac{a_w^2}{4\gamma_r^2}$$

$$\times \frac{\partial}{\partial J}\left\{ \left[\sum_{m=0}^{\infty} i\, m\, Q_m^{\pm}(J)\, a_s(q_s)\, e^{i[m\theta \pm \delta(q_s)z]} + cc\right] \right. \qquad \text{(C8)}$$

$$\left. \left[\sum_{n=0}^{\infty}\frac{n\, Q_n^{\pm}(J)\, a_s(k_s)}{n\, \kappa_b(J) \pm \delta(k_s)}\frac{\partial f_o}{\partial J} e^{i[n\theta \pm \delta(k_s)]} + cc\right]\right\},$$

where again $\delta(k_s) = (k_w/k_r)[(k_s - k_r) + i\varepsilon]$, $\varepsilon = \mathrm{Im}(k_s)$. Integration of the right-hand side of (C8) over θ yields,

$$\int_0^{2\pi}\frac{d\theta}{2\pi}\left\{\ldots\right\} = -\sum_{n=0}^{\infty}\left[i n^2 \frac{|Q_n^{\pm}|^2\, a_s^{*}(q_s)\, a_s(k_s)}{n\, \kappa_b(J) \pm \delta(k_s)} e^{\pm[\delta^{*}(q_s) - \delta(k_s)]z} + cc\right]. \qquad \text{(C9)}$$

Spatial integration yields

$$\frac{1}{L}\int_0^L dz\; a^*(q_s)\; a\,(k_s)\; e^{\pm\frac{k_w}{k_r}[(q_s - k_s) + 2\,i\,\varepsilon]z} = 2\pi\; W(q_s,z)\;\delta(q_s - k_s)\;, \tag{C10}$$

where $W_k(z) = a^2(k_s,z)/L$ is the spectral energy density. Substitution of the results (C9) and (C10) back in (C8) yields,

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial J}\, D_q(J)\, \frac{\partial f}{\partial J}\;,$$

where

$$D_q(J) = \frac{k_r}{4\pi}\,\frac{a_w^2}{\gamma_r^2}\,\frac{k_r}{k_w}\sum_{n=1}^{\infty}\int dk_s\, \frac{n^2 W_{k_s}(z)\;|Q_n^{\pm}|^2\;\varepsilon}{\left[\frac{k_r}{k_w}\, n\;\kappa_b(J) \pm (k_s - k_r)\right]^2 + \varepsilon^2}\,. \tag{C11}$$

In the limit of small growth rate $\varepsilon/k_s \ll 1$, (C11) is reduced to Eq. (42), Sec. IV,

$$D_q(J) = \frac{k_r}{4}\,\frac{k_r}{k_w}\,\frac{a_w^2}{\gamma_r^2}\sum_{n=0}^{\infty}|Q_n^{\pm}|^2\int_{-\infty}^{\infty} dk_s\; W_{k_s}(z)\;\delta(k_s - k_n)\;, \tag{C12}$$

where $k_n = k_r \pm 2\gamma_z^2 k_w$.

APPENDIX D: SUMMATION OF FOURIER COEFFICIENTS

We present a general technique of computing sums of the form,

$$\sum_{n=0}^{\infty} n^2 \left[|Q_n^{+}(J)|^2 + |Q_n^{-}(J)|^2 \right] . \tag{D1}$$

The quantities $Q_n^{\pm}(J)$ are the Fourier coefficients from the decomposition of the phase $\exp[i\psi(J,\theta)]$ of the perturbation into harmonics of the angle variable θ for the unperturbed system. The knowledge of the individual $Q_n^{\pm}(J)$ is not required in the computation. The technique should be applicable to a wide class of integrable systems experiencing a periodic perturbation with only minor modifications. In our case $Q_n^{\pm}(J)$ are defined by,

$$\cos\,[\psi(J,\theta) + \delta_s z] = \sum_{n=0}^{\infty} Q_n^{+}(J)\,\cos(n\theta + \delta_s z) + Q_n^{-}(J)\,\cos(n\theta - \delta_s z). \tag{D2}$$

Closed forms for $Q_n^{\pm}(J)$, obtained in Ref. 2 for the case of an untapered wiggler, appear in Appendix A.

For untrapped particles we have,

$$Q_{2m}^{\pm} = Q_{2m}\,, \qquad Q_{2m+1}^{\pm} = \pm\, Q_{2m+1}.$$

Setting $\delta_s = 0$ in (D2) and differentiating in θ, we obtain,

$$\frac{\partial}{\partial\theta}\cos\psi = -\frac{d\psi}{d\theta}\sin\psi = -\sum_{m=1}^{\infty}(2m)\;2Q_{2m}\,\sin 2m\theta\,, \tag{D3a}$$

$$\frac{\partial}{\partial\theta} \sin\,[\theta - \tfrac{\pi}{2}] = \frac{d\psi}{d\theta} \cos\, q = \sum_{m=0}^{\infty} (2m+1)\; 2Q_{2m+1}\; \cos[(2m+1)\theta]. \tag{D3b}$$

Squaring the right-hand sides of (D3a), (D3b), adding them and integrating over θ we obtain,

$$\sum n^2 \left[|Q_n{}^+|^2 \cdot |Q_n{}^-|^2\right] = 2 \sum n^2 |Q_n|^2 = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{d\psi}{d\theta}\right)^2 (\cos^2\psi + \sin^2\psi)\; d\theta \tag{D4}$$

Applying the same procedure to untrapped particles we obtain,

$$\frac{\partial}{\partial\theta} \cos\,\psi = - \frac{d\psi}{d\theta} \sin\,\psi = - \sum_{m=1}^{\infty} n\; (Q_n{}^+ + Q_n{}^-)\; \sin\, n\theta\; , \tag{D5a}$$

$$\frac{\partial}{\partial\theta} \sin\,[\theta - \tfrac{\pi}{2}] = \frac{d\psi}{d\theta} \cos\, q = - \sum_{m=0}^{\infty} n\; (Q_n{}^+ - Q_n{}^-)\; \cos\, n\theta\; , \tag{D5b}$$

and, after squaring, adding (D5a) and (D5b) and integrating over θ ,

$$\sum_{n=0}^{\infty} n^2 \left(|Q_n{}^+|^2 + |Q_n{}^-|^2\right) = \sum_{n=0}^{\infty} \frac{n^2}{2} \left(|Q_n{}^+ + Q_n{}^-|^2 + |Q_n{}^+ - Q_n{}^-|^2 \right)$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{d\psi}{d\theta}\right)^2 (\cos^2\psi + \sin^2\psi)\; d\theta\; . \tag{D6}$$

Thus in both cases,

$$\sum_{n=0}^{\infty} n^2 \left(|Q_n{}^+|^2 + |Q_n{}^-|^2\right) = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{d\psi}{d\theta}\right)^2 d\theta\; . \tag{D7}$$

Using the definition Eq. (11) for $\theta(\psi)$, and Eq. (8) for the unperturbed Hamiltonian ($a_s = 0$) in the right-hand side of (D7) we have,

$$\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{d\psi}{d\theta}\right)^2 d\theta = \frac{1}{2\pi}\, 2\int_{\psi_{mn}}^{\psi_{mx}} \frac{d\psi}{\left(\frac{d\theta}{d\psi}\right)}$$

$$= \frac{2\ k_w}{\gamma_r \omega_b(J)}\ \frac{1}{\pi}\int_{\psi_{mn}}^{\psi_{mx}} d\psi\ \left(\frac{\gamma_r H_o(J)}{k_w} - \frac{a_w a_r}{k_w}\left[\ \cos\psi + \psi\ \sin\psi_r\right]\right)^{1/2} . \tag{D8}$$

The last integral in (D8) is by the definition (11) the action J for the unperturbed Hamiltonian, yielding the final result,

$$\sum_{n=0}^{\infty} n^2\ \left(|Q_n{}^+|^2 + |Q_n{}^-|^2\right) = \frac{2\ k_w}{\gamma_r \omega_b(J)}\ J\ . \tag{D9}$$

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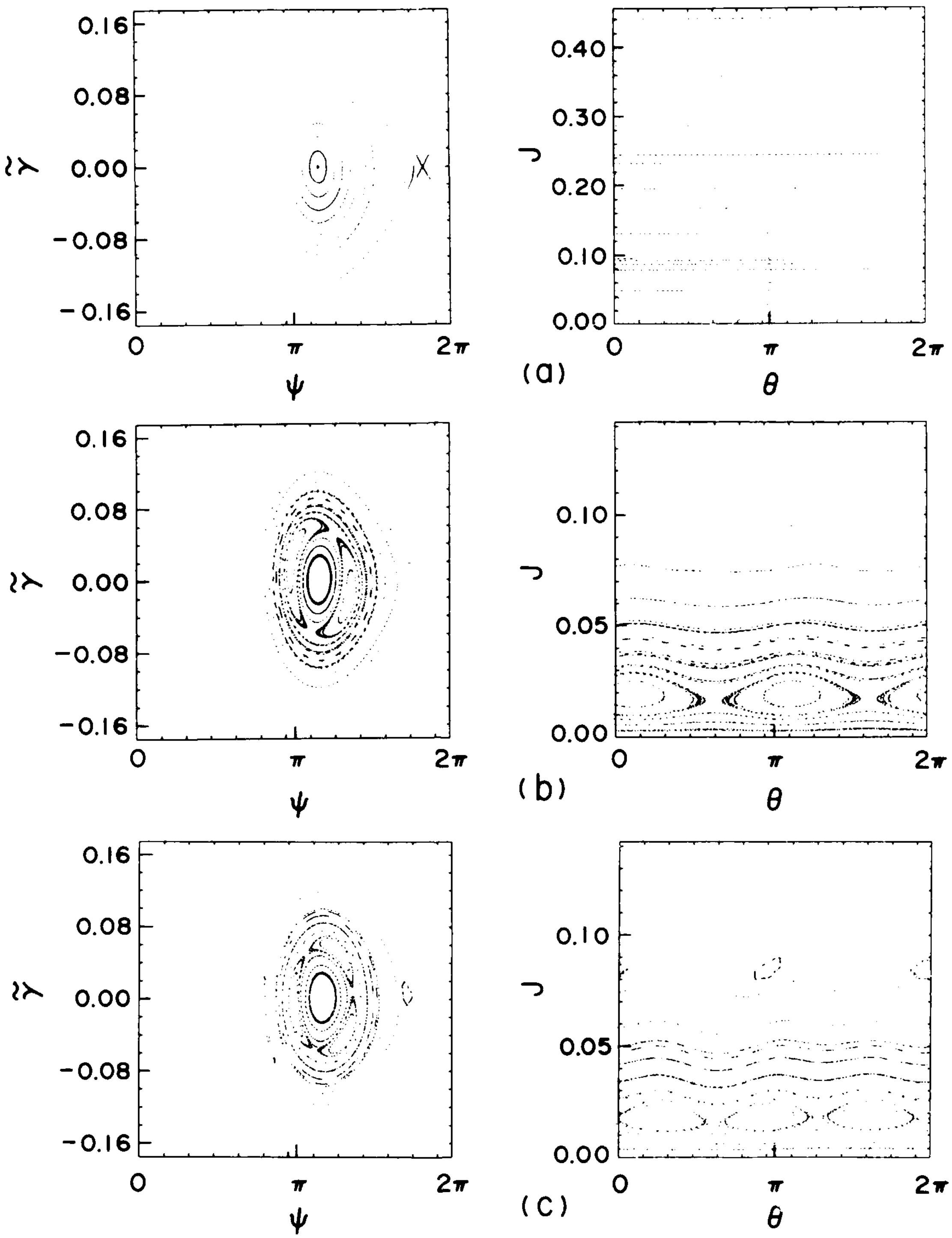

Figure 1. Surfaces of section expressed in γ, ψ coordinates on the left side and action-angle coordinates on the right side. The parameters are $a_w = 2$, $a_r = 5\times10^{-5}$, $\psi_r = 7\pi/6$, $\gamma_r = 25$ and (a) $a_s = 0$, (b) $a_s = 5\times10^{-7}$, $\omega_s/\omega_r = 1.016$, (c) $a_s = 2\times10^{-6}$, $\omega_s/\omega_r = 1.024$.

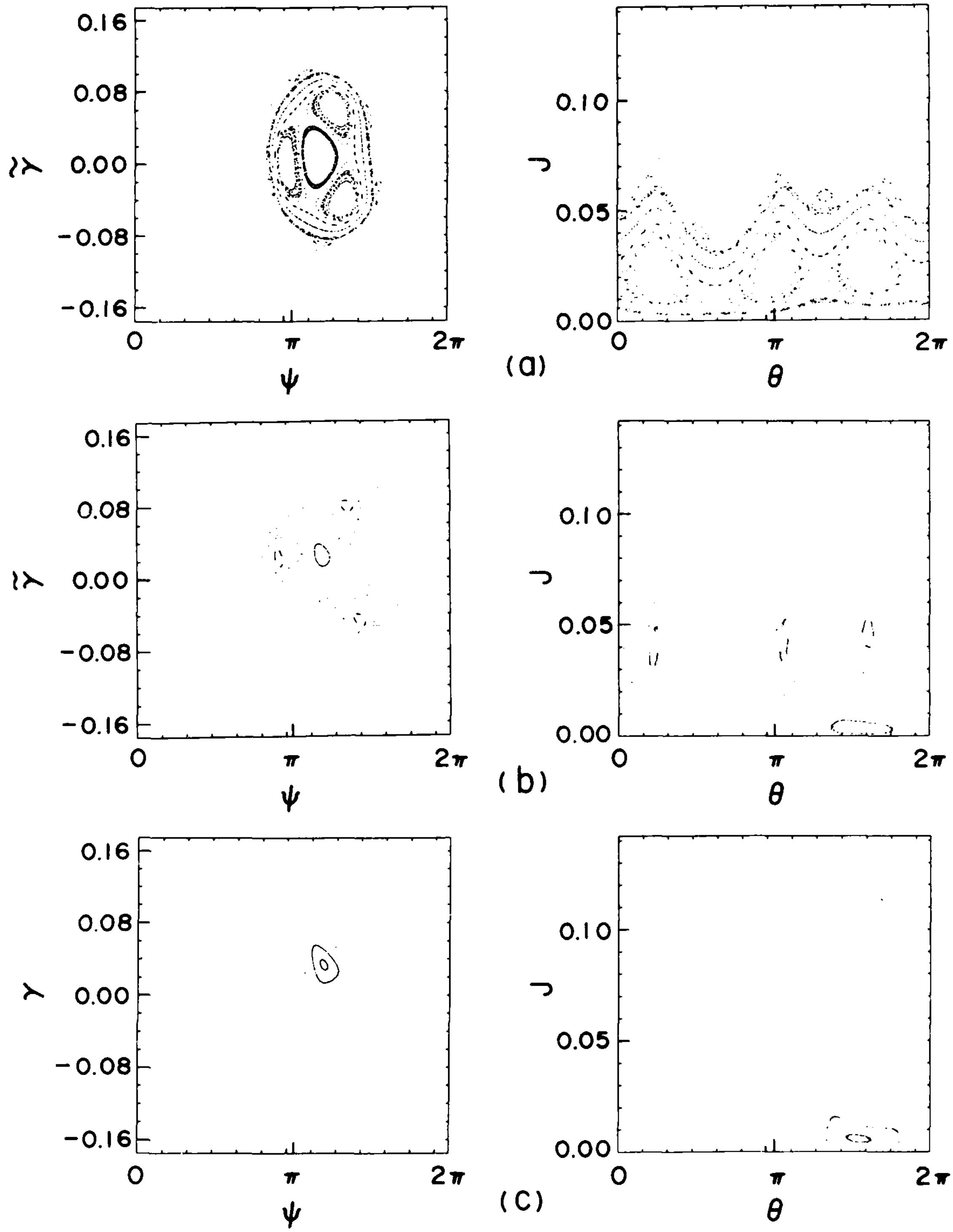

Figure 2. Transition to chaotic behavior. Plotted again are surfaces of section in both $\tilde{\gamma}$, ψ and J, θ representations. The parameters are $a_w = 2$, $a_r = 5\times10^{-5}$, $\psi_r = 7\pi/6$, $\gamma_r = 25$ and $\omega_s/\omega_r = 1.024$. The sideband amplitude increases from (a) $a_s = 1\times10^{-5}$ to (b) $a_s = 3\times10^{-5}$ to (c) $a_s = 5\times10^{-5}$.

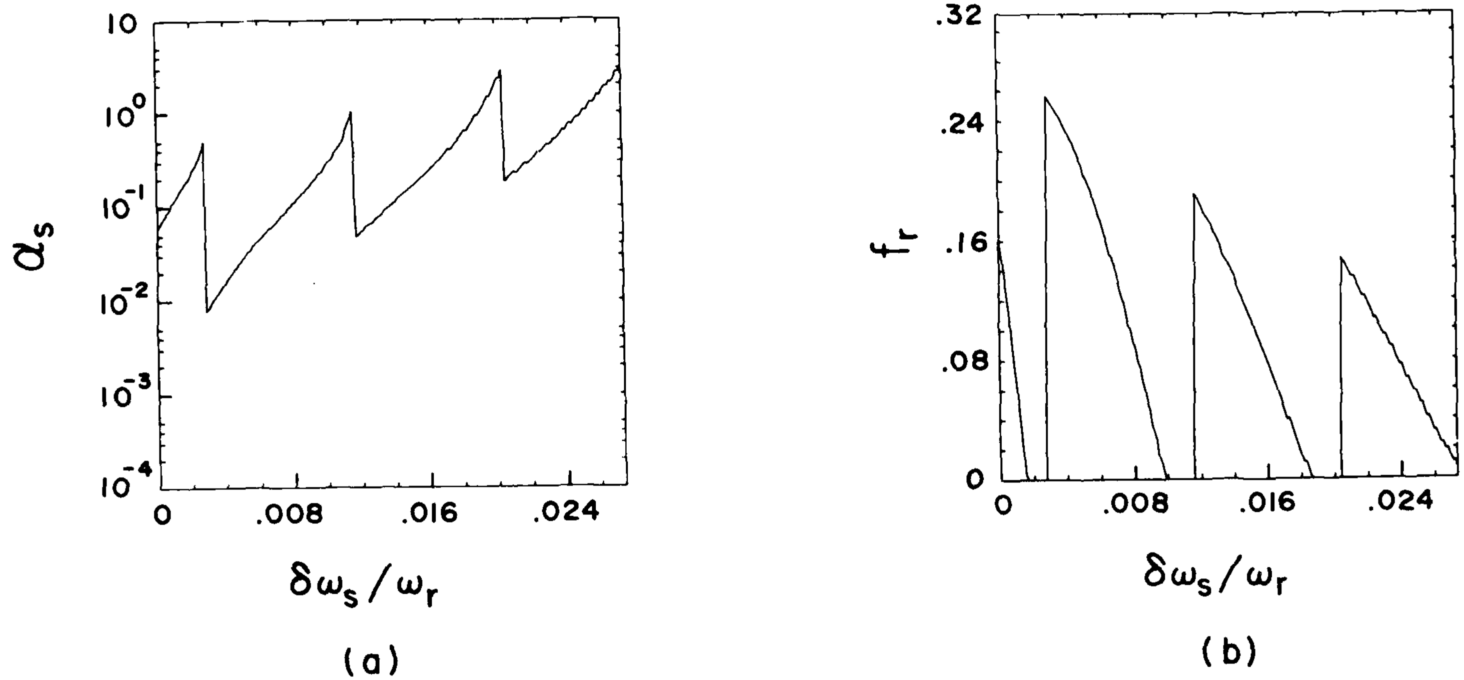

Figure 3. Stochasticity threshold α_s for single frequency sideband. (a) Plot of α_s against the sideband frequency mismatch $\delta\omega/\omega_r$ (solid line), signifying the overlapping of the two innermost secondary island chains (extended stochastic behavior). The dashed line α_s' signifies the overlapping of the next two islands on the outside (limited stochastic behavior). (b) The fraction f_s of the phase area inside the separatrix *that remains integrable when* $a_s = \alpha_s$ (solid) and when $a_s = \alpha_s'$ (dashed) line.

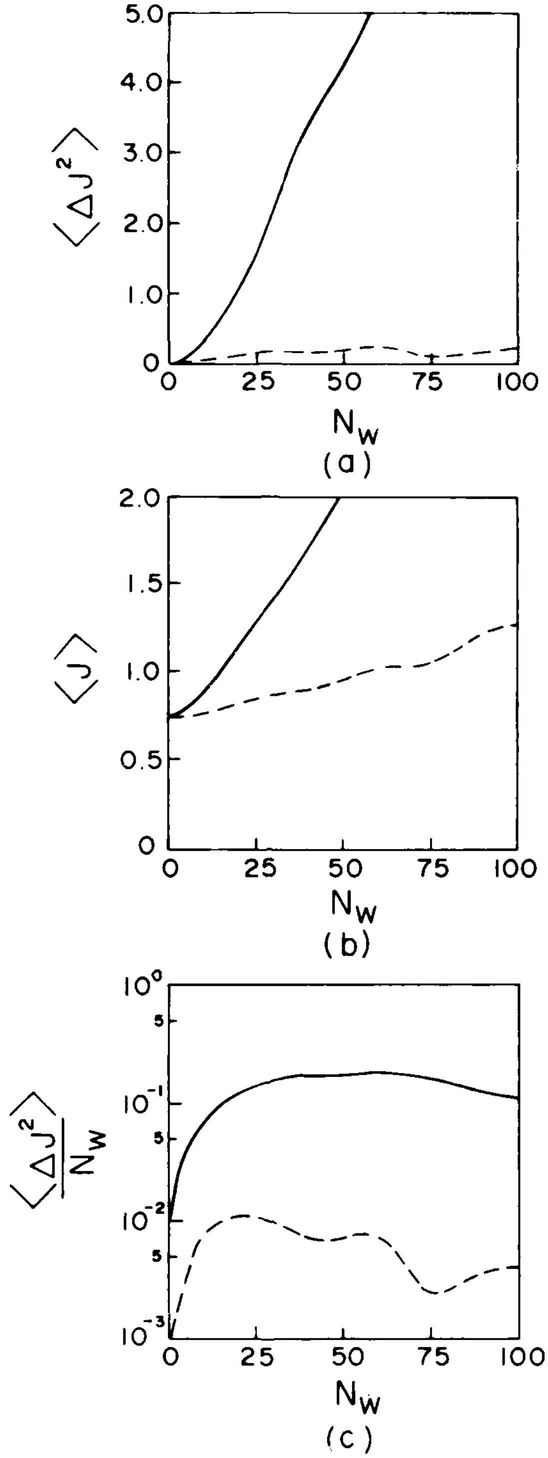

Figure 4. Diffusive behavior of a "monoenergetic" initial distribution $J_o = 0.7\ J_s$ with $a_w = 2$, $a_r = 5\times10^{-5}$, $\psi_r = 7\pi/6$, $\gamma_r = 25$ and $\omega_s/\omega_r = 1.016$. Plotted are (a) $< \Delta J^2 >$ (b) $< J >$ and (c) $< \Delta J^2 > / z$ as functions of $N_w = z / \lambda_w$. The solid curves correspond to $a_s = 1.5\times10^{-5}$ and the dashed ones to $a_s = 5\times10^{-5}$.

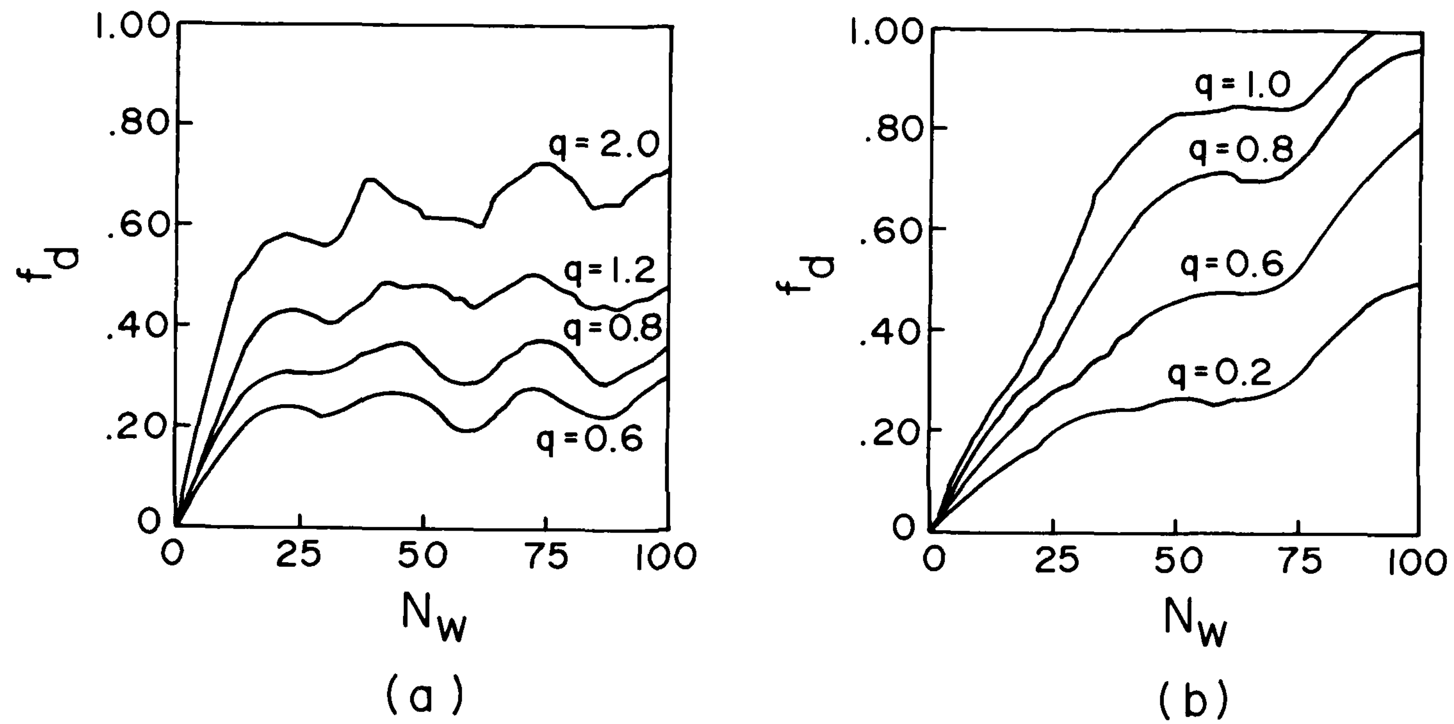

Figure 5. The fraction f_d of detrapped particles against the distance in wiggler periods $N_w = z / \lambda_w$. Different curves correspond to various sidebands to carrier amplitude ratios $q = a_s/a_r$. Parameters are $a_w = 2$, $a_r = 5\times10^{-5}$, $\psi_r = 7\pi/6$, $\gamma_r = 25$ and (a) $\omega_s/\omega_r = 1.016$, (b) $\omega_s/\omega_r = 1.024$.

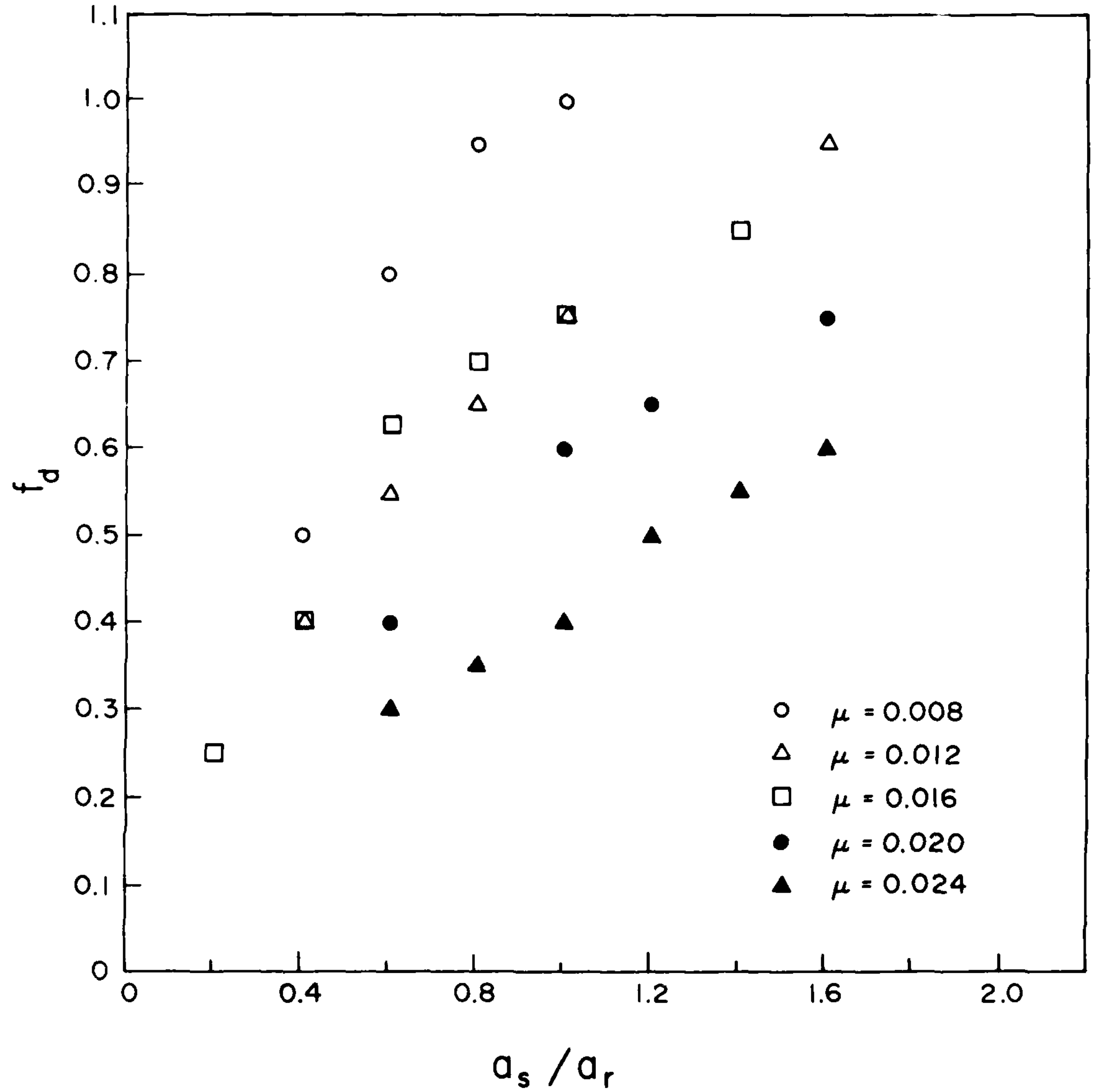

Figure 6. The fraction f_d of detrapped particles after 100 wiggler periods for an initially uniformly filled bucket. Results for various sideband frequency ratios $\mu = \omega_s/\omega_r$ are plotted against the relative sideband amplitude $q = a_s/a_r$ for $a_w = 2$, $a_r = 5\times10^{-5}$ and $\psi_r = 7\pi/6$, $\dot{\gamma}_r = 25$.

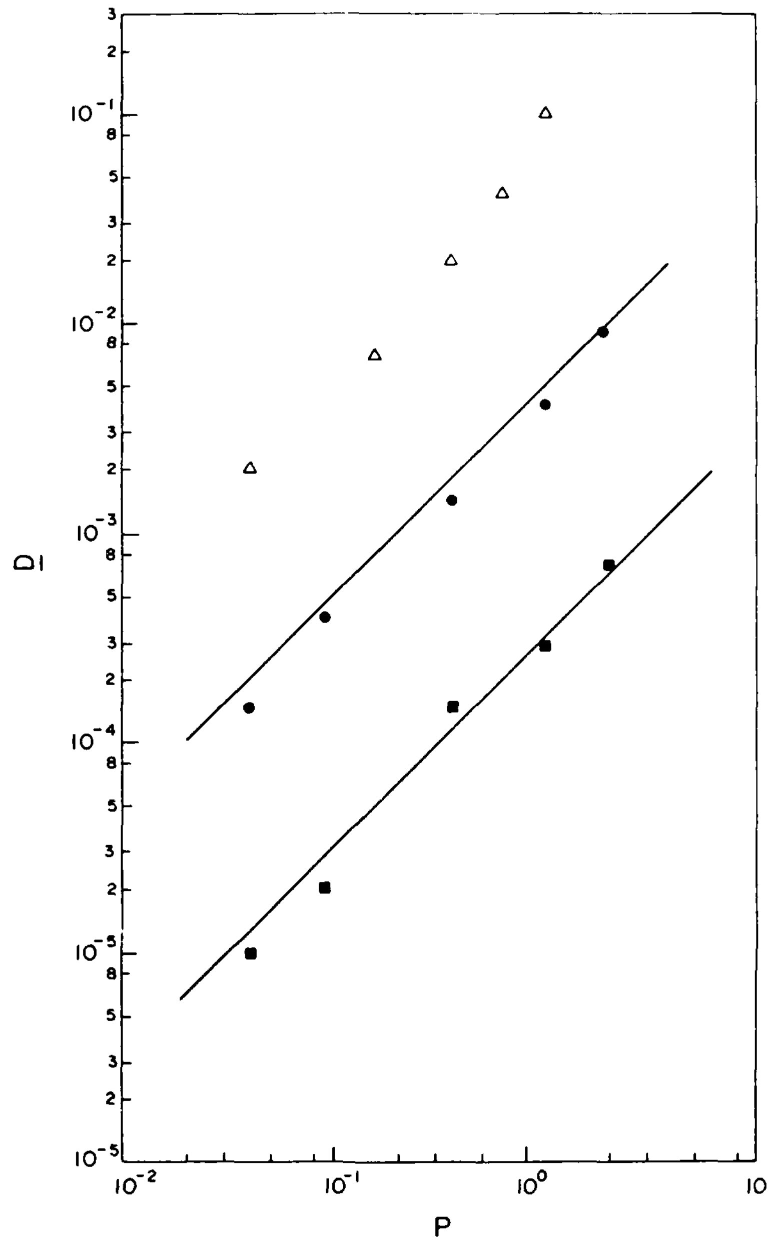

Figure 7. The normalized diffusion coefficient $\underline{D}$ as a function of the sideband power ratio $P = W_s/W_r$ for $a_w = 2$, $a_r = 5\times10^{-5}$ and $\psi_r = 7\pi/6$, $\gamma_r = 25$. Squares correspond to a continuous type of spectrum, peaked at $\omega_s/\omega_r = 1.016$ with $A = 100$ and $\nu = 0.1$. Dots correspond to a wide discrete peaked at $\omega_s/\omega_r = 1.024$ with $A = 20$ and $\nu = 0.5$. Triangles correspond to a single frequency spectrum with $\omega_s/\omega_r = 1.016$. The upper and lower solid lines correspond to the theoretical results from Eqs. (36) and (42) respectively.

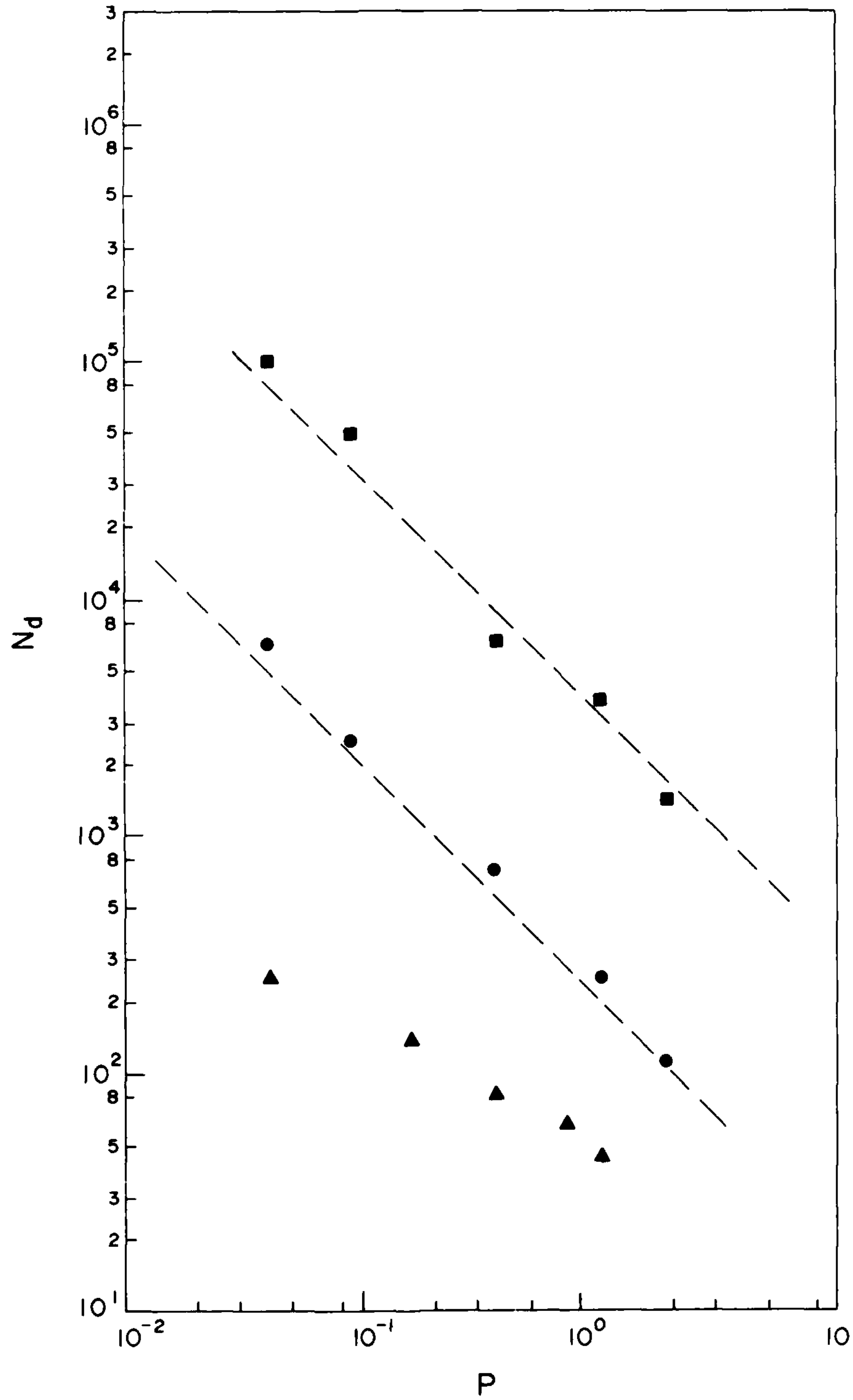

Figure 8. The e-folding diffusion length N_d in wiggler periods for the number of trapped particles. Parameters and semantics are the same as in Fig. 7.

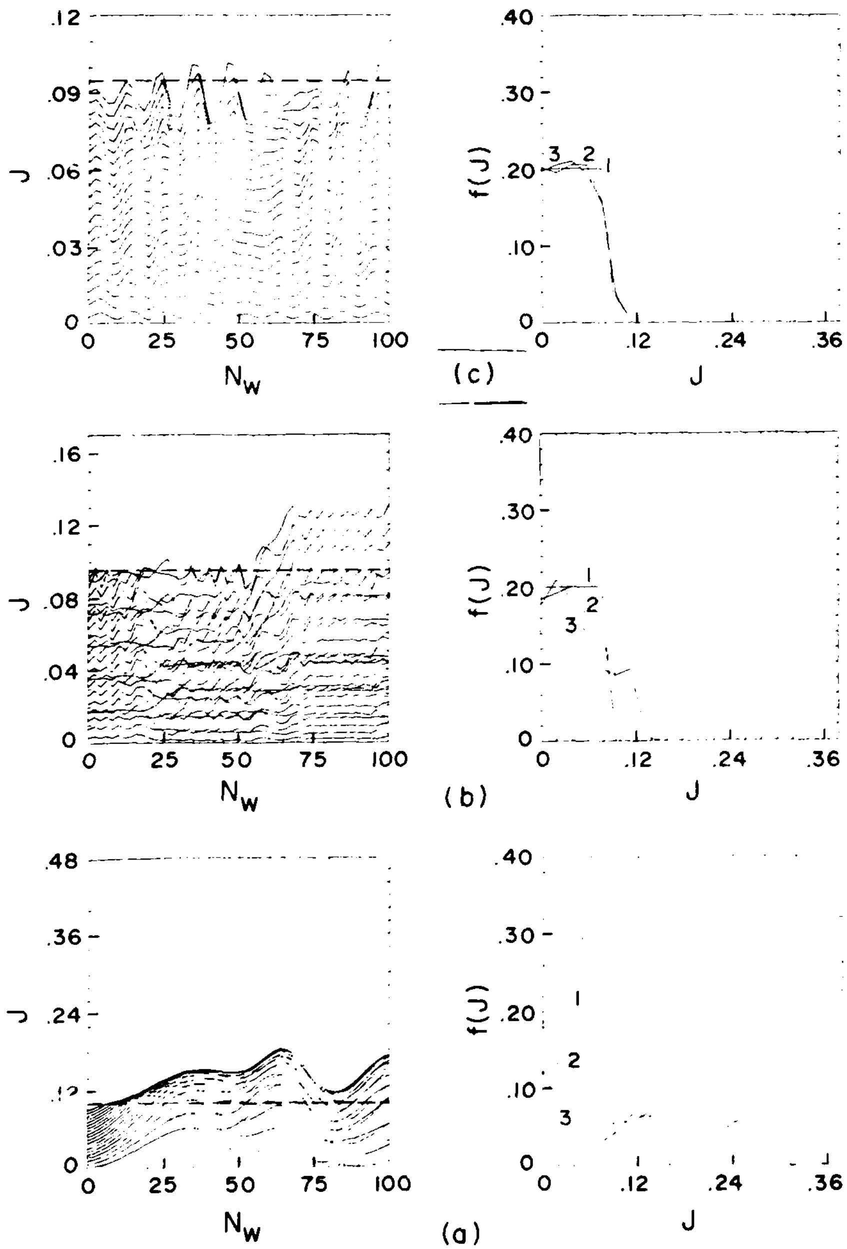

Figure 9. Particle response under different sideband spectra. On the left-hand side we plot the action J against the number of wiggler periods N_w for selected particles. On the right-hand side we plot the corresponding distribution function f(J) at N_w = 0, 50, and 100. In all cases the total sideband power ratio $P = W_s/W_r = 0.36$ and $a_w = 2$, $a_r = 5\times10^{-5}$, $\gamma_r = 25$. (a) corresponds to a wide continuous sideband spectrum (b) corresponds to a wide discrete spectrum and (c) to a single frequency $\omega_s/\omega_r = 1.016$.

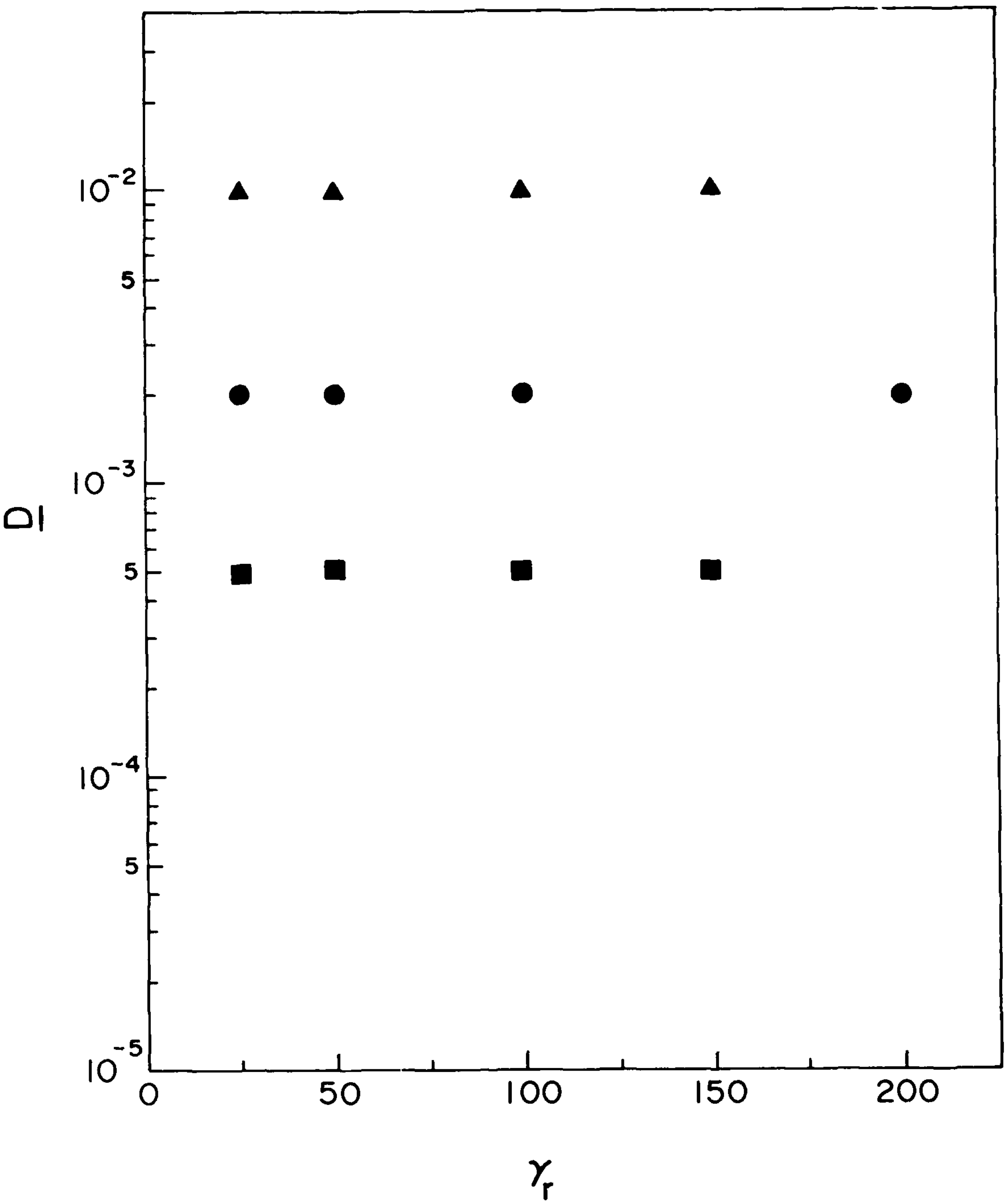

Figure 10. Plots of the normalized diffusion coefficient $\underline{D}$ against the electron energy γ_r. Parameters are $a_w = 2$, $a_r = 2 \times 10^{-4}$, $a_s = 7.5 \times 10^{-5}$ and $\omega_s = 1.024$. Squares correspond to a continuous, dots to a wide discrete and triangles to a single frequency spectrum.

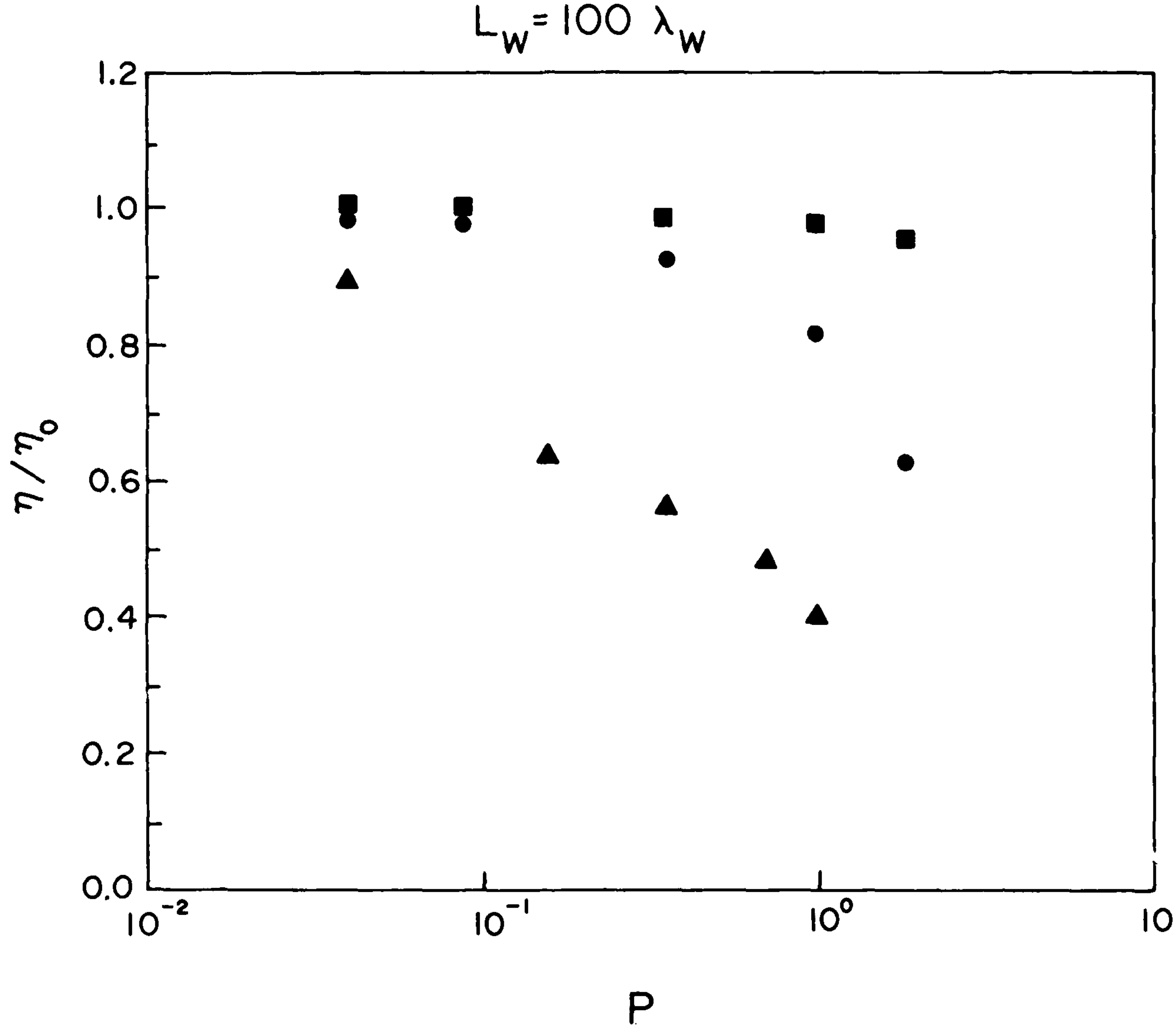

Figure 11. Deterioration of the extraction efficiency for various spectral types. Plotted is the ratio η/η_0 for a wiggler length of 100 periods as a function of the sideband power ratio P. The numerical results in Fig. 8 were used in Eq. (49) to create this plot. The symbolism and the parameters are the same as in Fig. 8.

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